Kunz, Ernst

Über die Klassifikation numerischer Halbgruppen. (On the classification of numerical semigroups). (German) Zbl 0618.14008

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This paper studies the numerical semigroups, i.e. subsets H of $\mathbb{N}$ such that $0 \in H$, $H + H \subseteq H$ and $\mathbb{N} - H$ is finite. It is well known that the study of numerical semigroups is closely related to the study of monomial curve singularities. In fact, for every numerical semigroup H one can associate the semigroup K-algebra $K[[H]] = \{\sum_{h \in H} a_h t^h | \quad a_h \in K\}$ (which is a K-subalgebra of K[[t]], where K is a fixed field). Many numerical invariants of the singularity K[[H]] such as the multiplicity, the embedding-dimension, the Hilbert function, etc. can be expressed in the language of semigroups. This point of view has been intensively studied by several authors e.g. Brezinski, Delorme, Herzog, Waldi, etc.

The author considers the set G_m of numerical semigroups which contain the fixed number $m \geq 3$, and associates to G_m a convex polyhedral cone $P_m \subset \mathbb{R}^{m-1}$ in such a way that there is a bijection between G_m and the set of points of P_m with integral coordinates. This method has the advantage of translating the problem of classification of the semigroups of G_m into a purely geometric language. In particular, one defines and studies the numerical invariant $s_m(H)$ which is defined as the dimension of the open face of P_m containing the corresponding integral point of P_m associated to H. This invariant is important because it gives a hierarchy of the semigroups of G_m. One also studies the Hilbert function of the integral points of a fixed face of P_m.

In an appendix one gives the complete lists concerning the classification of the semigroups of G_m with $m \leq 7$ (by using the invariants studied before and a computer program).

Reviewer: L.Bădescu

MSC:

14H20	Singularities of curves, local rings
13H15	Multiplicity theory and related topics
20M10	General structure theory for semigroups
14-04	Software, source code, etc. for problems pertaining to algebraic geometry

Keywords:

numerical semigroups; monomial curve singularities; multiplicity; embedding-dimension; Hilbert function; computer program