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Holomorphic families of injections. (English) Zbl 0619.30027
Acta Math. 157, 259-286 (1986).

This paper contains new proofs and extensions of some results by *R. Mañé*, *P. Sad* and *D. Sullivan* [*Ann. Sci. Éc. Norm. Supér.*, IV. Ser. 16, 193-217 (1983; [Zbl 0524.58025](#))] and *D. P. Sullivan* and *W. P. Thurston* [*Acta Math.* 157, 243-257 (1986; reviewed above)]. Let E be a subset of the Riemann sphere $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ containing at least 4 points. Let Δ_r denote the open disc $|z| < r$ in \mathbb{C} . A map

$$f : \Delta_r \times E \rightarrow \mathbb{C}$$

will be called admissible if $f(0, z) = z$ for all $z \in E$, for every fixed $\lambda \in \Delta_r$ the map $f(\lambda, \cdot) : E \rightarrow \hat{\mathbb{C}}$ is an injection, and for every fixed $z \in E$ the map $f(\cdot, z) : \Delta_r \rightarrow \hat{\mathbb{C}}$ is holomorphic.

Theorem 1. If $f : \Delta_1 \times E \rightarrow \hat{\mathbb{C}}$ is admissible, then every $f(\lambda, \cdot)$ is the restriction to E of a quasiconformal self-map F_λ of $\hat{\mathbb{C}}$, of dilatation not exceeding

$$K = (1 + |\lambda|)/(1 - |\lambda|).$$

Theorem 2. If $f : \Delta_1 \times E \rightarrow \mathbb{C}$ is admissible and E has a nonempty interior ω , then for each $\lambda \in \Delta_1$ the map $f(\lambda, \cdot)|_\omega$ is a K -quasiconformal homeomorphism of ω into $\hat{\mathbb{C}}$ with $K = (1 + |\lambda|)/(1 - |\lambda|)$. The Beltrami coefficient of $f(\lambda, \cdot)|_\omega$ given by

$$\mu(\lambda, z) = \frac{\partial f(\lambda, z)|_\omega}{\partial \bar{z}} / \frac{\partial f(\lambda, z)|_\omega}{\partial z}$$

is a holomorphic function of $\lambda \in \Delta_1$, and an element of the Banach space $L_\infty(\omega)$. The author's proofs make essential use of the theory of quasiconformal maps and of Teichmüller spaces.

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MSC:

[30C62](#) Quasiconformal mappings in the complex plane
[32G15](#) Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)

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References:

- [1] Agard, S. B. & Gehring, F. W., Angles and quasiconformal mappings. *Proc. London Math. Soc.* (3), 14 (1965), 1–21. · [Zbl 0131.07902](#) · [doi:10.1112/plms/s3-14A.1.1](#)
- [2] Ahlfors, L. V. & Bers, L., Riemann's mapping theorem for variable metrics. *Ann. of Math.* (2), 72 (1960), 385–404. · [Zbl 0104.29902](#) · [doi:10.2307/1970141](#)
- [3] Ahlfors, L. V. & Weill, G., A uniqueness theorem for Beltrami equations. *Proc. Amer. Math. Soc.*, 13 (1962), 975–978. · [Zbl 0106.28504](#) · [doi:10.1090/S0002-9939-1962-0148896-1](#)
- [4] Bers, L., Extremal quasiconformal mappings. *Advances in the Theory for Riemann Surfaces*, *Ann. of Math. Studies*, 66 (1970), 27–52.
- [5] –, Finite dimensional Teichmüller spaces and generalizations. *Bull. Amer. Math. Soc. (N.S.)*, 5 (1981), 131–172. · [Zbl 0485.30002](#) · [doi:10.1090/S0273-0979-1981-14933-8](#)
- [6] Earle, C. J. & Kra, I., On holomorphic mappings between Teichmüller spaces. *Contributions to Analysis*, Academic Press, New York, (1974), 107–124. · [Zbl 0307.32016](#)
- [7] Harvey, W. J. (Ed.), *Discrete groups and automorphic functions*. Academic Press, New York, 1977. · [Zbl 0411.30033](#)

- [8] Hille, E., *Analytic Function Theory, Vol. II.* Ginn and Co. (1962). · [Zbl 0102.29401](#)
- [9] Hubbard, J. H., Sur les sections analytique de la courbe universelle de Teichmüller. *Mem. Amer. Math. Soc.*, 166 (1976), 1–137. · [Zbl 0318.32020](#)
- [10] Lehto, O. & Virtanen, K. I., *Quasiconformal mappings in the plane.* Springer-Verlag, Berlin, 1973. · [Zbl 0267.30016](#)
- [11] Mañé, R., Sad, P. & Sullivan, D., On the dynamics of rational maps. *Ann. Sci. Ecole Norm. Sup.*, 16 (1983), 193–217. · [Zbl 0524.58025](#)
- [12] Reich, E. & Strebel, K., Extremal quasiconformal mappings with given boundary values. *Contributions to Analysis*, Academic Press, New York, (1974), 375–391. · [Zbl 0318.30022](#)
- [13] Royden, H. L., Automorphisms and isometries of Teichmüller space. *Advances in the Theory of Riemann Surfaces*, *Ann. of Math. Studies*, 66 (1971), 369–383.
- [14] Strebel, K., On quasiconformal mappings of open Riemann surfaces. *Comment. Math. Helv.*, 52 (1978), 301–321. · [Zbl 0421.30017](#) · [doi:10.1007/BF02566081](#)
- [15] Sullivan, D. & Thurston, W. P., Extending holomorphic motions. *Acta Math.*, 157 (1986), 243–257. · [Zbl 0619.30026](#) · [doi:10.1007/BF02392594](#)

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