Kuwert, Ernst; **Li, Yuxiang**

$W^{2,2}$-conformal immersions of a closed Riemann surface into $\mathbb{R}^n$. (English) Zbl 1271.53010
Commun. Anal. Geom. 20, No. 2, 313-340 (2012).

Summary: We study sequences $f_k : \Sigma_k \to \mathbb{R}^n$ of conformally immersed, compact Riemann surfaces with fixed genus and Willmore energy $\mathcal{W}(f) \leq \Lambda$. Assume that Σ_k converges to Σ in moduli space, i.e., $\phi_k^*(\Sigma_k) \to \Sigma$ as complex structures for diffeomorphisms ϕ_k. Then we construct a branched conformal immersion $f : \Sigma \to \mathbb{R}^n$ and Möbius transformations σ_k, such that, for a subsequence $\sigma_k \circ f_k \circ \phi_k \to f$ weakly in $W^{2,2}_{\text{loc}}$ away from finitely many points. For $\Lambda < 8\pi$ the map f is unbranched. If the Σ_k diverge in moduli space, then we show $\liminf_{k\to\infty} \mathcal{W}(f_k) \geq \min(8\pi, \omega_p^n)$. Our work generalizes results in one of our recent preprints to arbitrary codimension.

MSC:

53A30 Conformal differential geometry (MSC2010)
53A07 Higher-dimensional and -codimensional surfaces in Euclidean and related n-spaces
49Q10 Optimization of shapes other than minimal surfaces

Keywords:

conformally immersed; Willmore energy; Möbius transformations

Full Text: DOI arXiv