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A lemma on nearby cycles and its application to the tame Lubin-Tate space. (English)

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This paper gives a transparent and simple proof of the work of *T. Yoshida* [Adv. Stud. Pure Math. 58, 361–402 (2010; Zbl 1257.11103)] on the study of cohomology of tame Lubin-Tate space. The main tool used in this paper is a lemma on the nearby cycles for semi-stable schemes.

Let K be a finite extension of $\mathbb{Q}_p$ with ring of integers $\mathcal{O}$ and residue field $\mathbb{F}_q$. Fix an integer d. Let $\breve{K}_d$ denote the completion of the maximal unramified extension of K, adjoined with (q^d-1)st roots of a uniformizer of K. The Lubin-Tate space is the moduli space of the deformations of one-dimensional formal $\mathcal{O}$-modules of height d over $\overline{\mathbb{F}}_p$. One can add a tame level structure to the moduli problem and get a $\mathbb{F}_q^\times$-Galois cover $\mathcal{M}_1$ of $\mathcal{M}$. Yoshida constructed a semi-stable model of $\mathcal{M}_1$ over $\breve{K}_d$, whose special fiber admits a stratification so that an open stratum is isomorphic to a Deligne-Lusztig variety $\widetilde{\mathcal{X}}(w)$, and hence verify the tame local Langlands correspondence. General formalism of nearby cycles provides a morphism

$$R\Gamma(\mathcal{M}_{1,\mathbb{C}_p},\Lambda)\to R\Gamma(\widetilde{X}(w),\Lambda)$$

for $\Lambda=\mathbb{Z}/l^n\mathbb{Z}$ or $\mathbb{Z}_l$. However, the cohomology tool that Yoshida used only allows him to extract information on the level of alternating sums of the cohomology. The novelty of this paper is to prove a general lemma on nearby cycles of semistable schemes; from that the author easily deduces that the displayed morphism above is in fact an isomorphism (and by duality, he deduces the analogous isomorphism for the compactly supported cohomology). An immediate corollary of this result is that each $H^i_c(\mathcal{M}_{1,\mathbb{C}_p},\mathbb{Z}_l)$ is torsion-free.

The general result on nearby cycles for schemes with semi-stable reduction proved in this paper, generalizes previous results of Illusie and *W. Zheng* [Bull. Soc. Math. Fr. 136, No. 3, 465–503 (2008; Zbl 1216.14016)]. In rough terms, it states that the restriction of the nearby cycles complex to a closed stratum is the push-forward of its restriction to the corresponding open stratum. The main difference with Zheng's result is that the author allows generalized semi-stable models and deep strata.

This is a very nice and short paper, containing a lot of interesting results.

Reviewer: Liang Xiao (Irvine)

MSC:

11S37 Langlands-Weil conjectures, nonabelian class field theory
14F20 Étale and other Grothendieck topologies and (co)homologies
14G22 Rigid analytic geometry

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Lubin-Tate space; local Langlands correspondence; nearby cycles; Delgine-Lusztig varieties

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