

**Manin, Yu. I.; Tsfasman, M. A.**

**Rational varieties: Algebra, geometry and arithmetic.** (English. Russian original)

Zbl 0621.14029

Russ. Math. Surv. 41, No. 2, 51-116 (1986); translation from Usp. Mat. Nauk 41, No. 2(248), 43-94 (1986).

After the appearance of *Yu. I. Manin's* book "Cubic forms: Algebra, geometry, arithmetic" in 1972 [Moscow 1972; Zbl 0255.14002; see also the English translation (Amsterdam 1974; second edition 1986)] significant progress has been achieved in the theory of rational varieties and their diophantine properties. The present paper is an excellent survey in this classical area, which might be called "Thirteen years later".

Everywhere it is supposed that the ground field  $k$  is perfect. - In § 1 the rational curves are described. - The geometry of the rational surfaces (the field  $k$  is algebraically closed) is presented in § 2. - In § 3 the case when  $k$  is not algebraically closed is dealt with. - The authors describe some  $k$ -birational invariants of rational surfaces in § 4. - The problems of Lüroth, Zariski, Noether, and the three-dimensional geometry of varieties close to the rational ones, form the subject of the purely geometrical § 8. - Almost all the rest of the survey is devoted to arithmetical questions: descent theory in works of J.-L. Colliot-Thélène and J.-J. Sansuc (§ 5); the group of zero-cycles (§ 6); cubic surfaces of del Pezzo and Châtelet surfaces (§ 7); intersections of two quadrics (§ 9). - In § 10 many open problems are stated.

Reviewer: V.Iliev

**MSC:**

- 14M20 Rational and unirational varieties
- 14K25 Theta functions and abelian varieties
- 14H45 Special algebraic curves and curves of low genus

Cited in **3** Reviews  
Cited in **26** Documents

**Keywords:**

rational varieties; varieties close to the rational ones

**Full Text:** [DOI](#)