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Some backward uniqueness results. (English) Zbl 0622.35029
Nonlinear Anal., Theory Methods Appl. 10, 777-790 (1986).

This paper establishes backward uniqueness and unique continuation results for nonlinear parabolic equations and inequalities. There are two principal results. One establishes backward uniqueness for inequalities of the type

$$\|du/dt + \mu A(t)u(t)\|_H \leq n(t)\|u(t)\|_{D[A^{1/2}(t)]} \quad ,$$

where $\{A(t)\}_{t \geq 0}$ is a family of self-adjoint unbounded linear operators on a Hilbert space H and $n(t)$ is a square integrable function. The second result shows that for u not identically zero then there exist $a, b > 0$, such that $|u(t)| \geq C \exp[-Re\mu(1+b)at]$; this is the unique continuation result.

The results are applied to the Navier-Stokes equations, equations for a heat conducting viscous fluid, MHD equations, and the Kuramoto- Sivashinsky equations on both bounded and unbounded spatial domains; L^2 integrability is always assumed.

Reviewer: [B.Straughan](#)

MSC:

- [35K55](#) Nonlinear parabolic equations
- [35Q30](#) Navier-Stokes equations
- [35B60](#) Continuation and prolongation of solutions to PDEs
- [35A05](#) General existence and uniqueness theorems (PDE) (MSC2000)
- [34G10](#) Linear differential equations in abstract spaces
- [35B40](#) Asymptotic behavior of solutions to PDEs

Cited in **33** Documents

Keywords:

backward uniqueness; unique continuation; inequalities; self-adjoint unbounded linear operators; Hilbert space; Navier-Stokes equations; heat conducting viscous fluid; MHD equations; Kuramoto-Sivashinsky equations

Full Text: [DOI](#)

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