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**The impact of Gödel's incompleteness theorems on mathematics.** (English) Zbl 1288.03003

Baaz, Matthias (ed.) et al., Kurt Gödel and the foundations of mathematics. Horizons of truth. Cambridge: Cambridge University Press (ISBN 978-0-521-76144-4/hbk). 3-25 (2011).

This 25-pages book chapter discusses the mathematical effects of Gödel's incompleteness theorems. Not less than 10 pages of it constitute the Appendix: Modularity, Fermat's Last Theorem, and PA (Peano's arithmetic). The rest (main body) consists of 10 sections: 1. Gödel's contemporaries in logic, 2. The mathematical evolution of the ideas, 3. How the number theorists react to the Gödel phenomenon, 4. Group theory, 5. Geometry and dynamical systems, 6. Set theory, 7. Logical form, 8. Ramsey independence, 9. Topological modere, and finally 10. Conclusion. So, this chapter is historical, philosophical and mathematical in nature, discussing wide areas where Gödel's incompleteness phenomenon may show up. We read in the Conclusion that:

“As far as incompleteness is concerned, its remote presence has little effect on current mathematics. Some of the techniques that originated in Gödel's early work (and in the work of his contemporaries) remain central in logic and occasionally in work connecting logic and the rest of mathematics. The long-known connections between Diophantine equations, or combinatorics, and consistency statements in set theory seem to have little to do with major structural issues in arithmetic. That PA is entirely natural in the context of finite combinatorics can hardly be denied, but no one has succeeded in crossing the gap between finite combinatorics and arithmetic (especially arithmetic geometry). As far as the geometry of sufficiently general Riemannian manifolds is concerned, techniques descended from Gödel have proved illuminating (Soare, 2004), without the results being sold as ‘natural independence’.”

For the entire collection see [[Zbl 1253.00009](#)].

Reviewer: [Saeed Salehi \(Tabriz\)](#)

**MSC:**

- [03-03](#) History of mathematical logic and foundations
- [00A30](#) Philosophy of mathematics
- [03F40](#) Gödel numberings and issues of incompleteness
- [00A35](#) Methodology of mathematics
- [01A65](#) Development of contemporary mathematics
- [03A05](#) Philosophical and critical aspects of logic and foundations
- [03B25](#) Decidability of theories and sets of sentences
- [11U05](#) Decidability (number-theoretic aspects)

Cited in 1 Document

**Keywords:**

[Gödel's incompleteness theorems](#); [philosophy of mathematics](#); [logical number theory](#); [arithmetic](#)