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Polytopes of minimum positive semidefinite rank. (English) Zbl 1279.52023

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Summary: The positive semidefinite (psd) rank of a polytope is the smallest k for which the cone of $k \times k$ real symmetric psd matrices admits an affine slice that projects onto the polytope. In this paper we show that the psd rank of a polytope is at least the dimension of the polytope plus one, and we characterize those polytopes whose psd rank equals this lower bound. We give several classes of polytopes that achieve the minimum possible psd rank including a complete characterization in dimensions two and three.

MSC:

52C45 Combinatorial complexity of geometric structures
52B11 n-dimensional polytopes

Keywords:

positive semidefinite rank; polytope; slack matrix; Hadamard square roots; cone lift

Software:

Macaulay2

Full Text: DOI arXiv

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.