Kaplan, Eyal

Multiplicativity of the gamma factors of Rankin-Selberg integrals for $\mathrm{SO}_{2l} \times \mathrm{GL}_n$. (English)

Zbl 1318.11158
Manuscr. Math. 142, No. 3-4, 307-346 (2013).

Let F be a p-adic field. For positive integers l and n, let H be a quasisplit form of the special orthogonal group SO_{2l}, and let $G = \mathrm{GL}_n$. Let π and τ be irreducible smooth generic representations of $H(F)$ and $G(F)$, respectively. The article under review studies the Rankin-Selberg integrals attached to the pair π, τ by *S. Gelbart* et al. [Explicit constructions of automorphic L-functions. Berlin: Springer (1987; Zbl 0612.10022)] and *D. Ginzburg* [J. Reine Angew. Math. 405, 156–180 (1990; Zbl 0684.22009)]. These integrals can be used to define the γ-factor $\Gamma(\pi \times \tau, \psi, s)$, where ψ is a fixed additive character of F.

The author proves that the association $(\pi, \tau) \rightsquigarrow \Gamma(\pi \times \tau, \psi, s)$ behaves multiplicatively in the arguments π and τ with respect to parabolic induction. Using archimedean results of *D. Soudry* [Ann. Sci. Éc. Norm. Supér. (4) 28, No. 2, 161–224 (1995; Zbl 0824.11034)] and global arguments, the author then shows that the function $\Gamma(\pi \times \tau, \psi, s)$ coincides with the standard γ-factor defined by Shahidi. The results of this paper have been used to show that the local L-factor attached to π, τ (in the tempered case) defined via Rankin-Selberg integrals coincides with the L-function $L(\pi \times \tau, s)$ defined by Shahidi.

Reviewer: Joshua Lansky (Washington)

MSC:

11S40 Zeta functions and L-functions
11F70 Representation-theoretic methods; automorphic representations over local and global fields
22E50 Representations of Lie and linear algebraic groups over local fields

Keywords:

gamma factor; Rankin-Selberg integral

Full Text: DOI

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