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Bounds for sectional genera of varieties invariant under Pfaff fields. (English) Zbl 1281.32027
Ill. J. Math. 56, No. 2, 343-352 (2012).

The abstract says: "We establish an upper bound for the sectional genus of varieties which are invariant under Pfaff fields on projective spaces". A more detailed summary should at least (assuming the reader is familiar with the nomenclature) include the statement of their results:

Theorem. Let X be a nonsingular projective variety of dimension m which is invariant under a Pfaff field \mathcal{F} of rank k on \mathbb{P}^n ; assume that $m \geq k$. If the tangent bundle Θ_X is stable, then

$$\frac{2g(X, \mathcal{O}_X(1)) - 2}{\deg(X)} \leq \frac{\deg(\mathcal{F}) - k}{\binom{m-1}{k-1}} + m - 1.$$

Theorem. Let $X \subset \mathbb{P}^n$ be a Gorenstein projective variety nonsingular in codimension 1, which is invariant under a Pfaff field \mathcal{F} on \mathbb{P}^n whose rank is equal to the dimension of X . Then

$$\frac{2g(X, \mathcal{O}_X(1)) - 2}{\deg(X)} \leq \deg(\mathcal{F}) - 1.$$

This result generalizes the bounds in [A. Campillo et al., J. Lond. Math. Soc., II. Ser. 62, No. 1, 56-70 (2000; Zbl 1040.32027)].

Reviewer: [Pedro Fortuny Ayuso \(Gijón\)](#)

MSC:

32S65 Singularities of holomorphic vector fields and foliations
37F75 Dynamical aspects of holomorphic foliations and vector fields
58A17 Pfaffian systems

Cited in **2** Documents

Keywords:

[Poincaré problem](#); [sectional genus](#); [Pfaff field](#); [invariant varitey](#)

Full Text: [Euclid](#)

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