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On the Kodaira dimension of minimal threefolds. (English) Zbl 0625.14023

Math. Ann. 281, No. 2, 325-332 (1988).

We show that a complex threefold X has Kodaira dimension ≥ 0 if X admits a minimal model. In view of a recent result of *S. Mori* ["Flip theorem and the existence of minimal models for 3-folds", J. Am. Math. Soc. 1 (1988), to appear], our theorem amounts to the following characterization of threefolds of Kodaira dimension $-\infty$: For a complex projective threefold X, $\kappa(X) = -\infty$ if and only if X is uniruled.

The proof is a combination of algebro-geometric results (the pseudo- effectivity of c_2 and the generic semi-positivity of Ω^1_X) and the differential geometric one (Donaldson's characterization of flat vector bundles on an algebraic surface).

MSC:

14J30 3-folds
14E30 Minimal model program (Mori theory, extremal rays)
14J25 Special surfaces

Keywords:

minimal model; characterization of threefolds; Kodaira dimension

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