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Approximation of solutions to second order nonlinear Picard problems with Carathéodory right-hand side. (English) Zbl 1297.34017

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A method to approximate the solution of Picard second order two-point boundary value problems (BVP's) with Carathéodory right-hand side is presented. This method is based on the idea to replace the right-hand side by its Kantorovich polynomial, obtaining a sequence of another BVPs that approximate the original BVP. It is proved that if the original BVP has at least one essential solution, then there exists a sequence of approximate solutions converging to this solution. The approximated BVPs are transformed into finite dimensional problems such that the approximate solutions are expressed by using piecewise constant functions. Consequently, the solution of the original BVP is the limit of a sequence of piecewise constant functions.

Reviewer: Alexandru Mihai Bica (Oradea)

MSC:

34A45 Theoretical approximation of solutions to ordinary differential equations
65L10 Numerical solution of boundary value problems involving ordinary differential equations
34B24 Sturm-Liouville theory

Keywords:

Picard problem; Kantorovich polynomial approximation; approximation of BVP solution

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