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On generalized Pólya urn models. (English) Zbl 1290.60009

J. Appl. Probab. 50, No. 4, 1169-1186 (2013).

Summary: We study an urn model introduced in the paper of *M. Chen* and *C. Wei* [J. Appl. Probab. 42, No. 4, 964–976 (2005; Zbl 1093.60007)], where at each discrete time step m balls are drawn at random from the urn containing colors white and black. Balls are added to the urn according to the inspected colors, generalizing the well known Pólya-Eggenberger urn model, case $m = 1$. We provide exact expressions for the expectation and the variance of the number of white balls after n draws, and determine the structure of higher moments. Moreover, we discuss extensions to more than two colors. Furthermore, we introduce and discuss a new urn model where the sampling of the m balls is carried out in a step-by-step fashion, and also introduce a generalized Friedman's urn model.

MSC:

60C05 Combinatorial probability
60F05 Central limit and other weak theorems
05A15 Exact enumeration problems, generating functions

Keywords:

urn model; limiting distribution

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References:

[1] Bagchi, A. and Pal, A. K. (1985). Asymptotic normality in the generalized Pólya-Eggenberger urn model, with an application to computer data structures. SIAM J. Algebraic Discrete Math. 6 , 394-405. · Zbl 0568.60010 · doi:10.1137/0606041

[2] Chen, M.-R. and Wei, C.-Z. (2005). A new urn model. J. Appl. Prob. 42 , 964-976. · Zbl 1093.60007 · doi:10.1239/jap/1134587809 ·

[3] Chen, M.-R., Hsiau, S.-R. and Yang, T.-H. (2012). A new two-urn model. To appear in J. Appl. Prob. · Zbl 1296.60071

[4] Eggenberger, F. and Pólya, G. (1923). Über die Statistik verketteter Vorgänge. Z. Angewandte Math. Mech. 1 , 279-289. · Zbl 49.0382.01

[5] Flajolet, P., Dumas, P. and Puyhaubert, V. (2006). Some exactly solvable models of urn process theory. In Discrete Mathematics and Theoretical Computer Science (Proc. Fourth Colloquium Math. Comp. Sci.), ed. P. Chassaing, pp. 59-118. · Zbl 1193.60011 · www.dmtcs.org

[6] Gouet, R. (1989). A martingale approach to strong convergence in a generalized Pólya-Eggenberger urn model. Statist. Prob. Lett. 8 , 225-228. · Zbl 0686.60023 · doi:10.1016/0167-7152(89)90126-0

[7] Gouet, R. (1993). Martingale functional central limit theorems for a generalized Pólya urn. Ann. Prob. 21 , 1624-1639. · Zbl 0788.60044 · doi:10.1214/aop/1176989134 ·

[8] Graham, R. L., Knuth, D. E. and Patashnik, O. (1994). Concrete Mathematics . Addison-Wesley. · Zbl 0836.00001

[9] Hill, B., Lane, D. and Sudderth, W. (1980). A strong law for some generalized urn processes. Ann. Prob. 8 , 214-226. · Zbl 0429.60021 · doi:10.1214/aop/1176994772 ·

[10] Janson, S. (2006). Limit theorems for triangular urn schemes. Prob. Theory Relat. Fields 134 , 417-452. · Zbl 1112.60012 · doi:10.1007/s00440-005-0442-7

[11] Johnson, N. L. and Kotz, S. (1977). Urn Models and Their Application . John Wiley, New York. · Zbl 0352.60001

[12] Kotz, S. and Balakrishnan, N. (1997). Advances in urn models during the past two decades. In Advances in Combinatorial Methods and Applications to Probability and Statistics , Birkhäuser, Boston, MA, pp. 203-257. · Zbl 0888.60014 · doi:10.1007/978-1-4612-4140-9_14

[13] Mahmoud, H. (2009). Pólya Urn Models . CRC Press, Boca Raton. · Zbl 1149.60005

[14] Pemantle, R. (1990). A time-dependent version of Pólya's urn. J. Theoret. Prob. 3 , 627-637. · Zbl 0708.60015 · doi:10.1007/BF01046101

[15] Renlund, H. (2010). Generalized Pólya urns via stochastic approximation. Availaible at http://arxiv.org/ abs/1002.3716.

[16] Schreiber, S. J. (2001). Urn models, replicator processes, and random genetic drift. SIAM J. Appl. Math. 61 , 2148-2167. · Zbl 0993.92026 · doi:10.1137/S0036139999352857