**Barbu, Dorel**; **Blot, Joel**; **Buse, Constantin**; **Saierli, Olivia**

**Stability for trajectories of periodic evolution families in Hilbert spaces.** (English)

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Electron. J. Differ. Equ. 2014, Paper No. 01, 13 p. (2014).

The authors consider a $q$-periodic evolution family $\mathcal{U}$ with propagator $A$ on a Hilbert space $H$. Under the assumptions that

(A) the trajectories $\mathcal{U}(\cdot,0)x$ for $x$ belonging to a dense subspace $D$ of $H$ satisfy a Lipschitz condition on $]0,q[$ and

(B) the solutions of $\dot{u}(t) = A(t)u(t) + \exp(i\mu t)x$ are uniformly bounded in $\mu \in \mathbb{R}$ and $x \in D$ with $\|x\| \leq 1$,

they derive a discrete boundedness condition for $\mathcal{U}$. The proof is based on Fourier-expansions of the functions involved and suitable estimates for their Fourier coefficients. The method of proof restricts the results to the Hilbert space case. It is remarkable that the boundedness condition for $\mathcal{U}$ implies the strong stability of the trajectories $\mathcal{U}(t,\cdot)x$ for $x \in D$ and, if (A) is assumed for all $x$ in $H$, the exponential stability of $\mathcal{U}$. Similar results are obtained in the autonomous case, i.e., when $\mathcal{U}(t,s) = T(t-s)$ for a $C_0$-semigroup $T$. In fact, in this case the condition (B) can be weakened by assuming the uniform boundedness just for $x$ in the unit ball of the domain of $A$, equipped with its graph norm.

Reviewer: Sascha Trostorff (Dresden)

**MSC:**

47D06 One-parameter semigroups and linear evolution equations
35B15 Almost and pseudo-almost periodic solutions to PDEs
35B10 Periodic solutions to PDEs



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