Mollin, R. A.; Williams, H. C.

**On prime valued polynomials and class numbers of real quadratic fields.** (English)

Zbl 0629.12004
Nagoya Math. J. 112, 143-151 (1988).

For an arbitrary positive square-free integer d we provide three sufficient conditions for the class number h(d) of $\mathbb{Q}(\sqrt{d})$ to equal 1. Under a certain hypothesis these conditions are shown to be necessary and sufficient. One of the conditions is that if $f_d(x) = -x^2 + x + (d-1)/4$ when $d \equiv 1 \pmod 4$, and $f_d(x) = d - x^2$ when $d \not\equiv 1 \pmod 4$, then $f_d(x)$ is prime for all integers x such that $1 < x < \alpha$, where $\alpha = \sqrt{d-1}/2$ when $d \equiv 1 \pmod 4$, and $\alpha = \sqrt{d}$ if $d \not\equiv 1 \pmod 4$.

Under the assumption of the generalized Riemann hypothesis (G.R.H.) we establish that each of the three conditions is equivalent to $h(d) = 1$ if and only if d is one of the 19 values below. The latter result establishes (modulo G.R.H.) conjectures of S. Chowla, R. Mollin, and H. Yokoi. A consequence of our main result (modulo G.R.H.) is that $f_d(x)$ is prime for all integers x with $1 < x < \alpha$ if and only if

$$d \in \{2, 3, 5, 6, 7, 11, 13, 17,^2 1,^2 9, 37, 53, 77, 101, 173, 197,^2 93, 437, 677\} .$$

This may be viewed as a general analog of the well-known Rabinovitch result for complex quadratic fields.

**MSC:**

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| 11R29 | Class numbers, class groups, discriminants |
| 11R11 | Quadratic extensions |
| 11R09 | Polynomials (irreducibility, etc.) |
| 11N32 | Primes represented by polynomials; other multiplicative structures of polynomial values |



**Keywords:**

real quadratic fields; prime producing polynomials; generalized Riemann hypothesis; class number one problem

**Full Text:** DOI

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