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Long-time behavior for the nonstationary Navier-Stokes flows in $L^1(\mathbb{R}^n_+)$. (English)

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Summary: The large time decay for the Navier-Stokes flows in $L^1(\mathbb{R}^n_+)$ is a long-standing unsolved question. The main difficulties are that: usual $L^q - L^r$ estimates for the Stokes flow fail in this case; and the projection operator $P : L^1(\mathbb{R}^n_+) \to L^1_\sigma(\mathbb{R}^n_+)$ becomes unbounded. Using the Stokes solution formula, we find a crucial and new estimate for the Stokes flow in $L^1(\mathbb{R}^n_+)$, which plays a fundamental role in studying the time L^1-behavior for the Navier-Stokes equations. In addition, we decompose the operator P into two parts, and reduce its unboundedness to establish an L^1 estimate for an elliptic problem with Neumann boundary condition, which is overcome by using the weighted estimates of the Gaussian kernel's convolution. The main results in this article are motivated by *H.-O. Bae*'s works [J. Differ. Equations 222, No. 1, 1–20 (2006; Zbl 1091.35055); J. Math. Fluid Mech. 10, No. 4, 503–530 (2008; Zbl 1188.35127)], and *L. Brandolese*'s work [C. R. Acad. Sci., Paris, Sér. I, Math. 332, No. 2, 125–130 (2001; Zbl 0973.35149)].

MSC:

35Q30 Navier-Stokes equations
76D05 Navier-Stokes equations for incompressible viscous fluids
76D07 Stokes and related (Oseen, etc.) flows
35D35 Strong solutions to PDEs

Keywords:

Navier-Stokes flows; solution formula; long-time behavior; strong solution

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