G. Labelle has proved that
\[ |a_n| \leq \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n - 1)}{n!} \left( \frac{2n + 1}{2} \right)^{1/2} \left( \int_{-1}^{1} |p_n(x)|^2 \, dx \right)^{1/2} \]
where \( p_n(x) = \sum_{\nu=0}^{n} a_{\nu} x^\nu \) is a polynomial of degree \( n \). The author improves this result by proving the following corresponding result: If \( p_n(x) = \sum_{\nu=0}^{n} a_{\nu} x^\nu \) is a polynomial of degree \( n \) such that \( p_n(1) = 0 \), then
\[ |a_n| \leq \frac{n}{n + 1} \frac{(2n)!}{2^n (n!)^2} \left( \frac{2n + 1}{2} \right)^{1/2} \left( \int_{-1}^{1} |p_n(x)|^2 \, dx \right)^{1/2}. \]
This inequality is sharp and the author gives the condition under which equality holds.

Reviewer: R.N. Siddiqi

MSC:
26C05 Real polynomials: analytic properties, etc.
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polynomials on the unit interval; coefficient estimates; Chebyshev polynomials; Legendre polynomials

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