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A mathematical analysis of fractional fragmentation dynamics with growth. (English)

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Summary: We make use of the theory of strongly continuous solution operators for fractional models together with the subordination principle for fractional evolution equations [*E. Bazhlekova*, Fract. Calc. Appl. Anal. 3, No. 3, 213–230 (2000; Zbl 1041.34046)] and [*J. Prüss*, Evolutionary integral equations and applications. Basel: Birkhäuser Verlag (1993; Zbl 0784.45006)] to analyze and show existence results for a fractional fragmentation model with growth characterized by its growth rate r. Indeed, strange phenomena like the phenomenon of shattering [*E. D. Grady* and *R. M. Ziff*, "Shattering" transition in fragmentation. Phys. Rev. Lett. 58, No. 9, 892–895 (1987)] and the sudden appearance of infinite number of particles in some systems with initial finite particles number could not be fully explained by classical models of fragmentation or aggregation. Then, there is an increasing volition to try new approaches and extend classical models to fractional ones. In the growth model, one of the major challenges in the analysis occurs when $1/r(x)$ is integrable at $X_0 \geq 0$, the minimum size of a cell. We restrict our analysis to the case of integrability of r^{-1} at x_0. This case needs more considerations on the boundary condition, which, in this paper, is the McKendrick-von Foerster renewal condition. In the process, some properties of Mittag-Leffler relaxation function [*M. N. Berberan-Santos*, J. Math. Chem. 38, No. 4, 629–635 (2005; Zbl 1101.33015)] are exploited to finally prove that there is a positive solution operator to the full model.

MSC:

35R11 Fractional partial differential equations
33E12 Mittag-Leffler functions and generalizations

Cited in **7** Documents

Full Text: DOI

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