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Polynômes de Bernstein-Sato à plusieurs variables. (Bernstein-Sato polynomials in several complex variables). (French) Zbl 0634.32003

Sémin., Équations Dériv. Partielles 1986-1987, Exp. No. 19, 6 p. (1987).

This paper is a summary of two papers of the author ["Proximité évanescente. I, II.", Compos. Math. 62, 283-328 (1987; Zbl 0622.32012), ibid. 64, 213-241 (1987; Zbl 0632.32006)]. The author first recalls the existence of the Bernstein-Sato polynomial for an analytic complex function or for a polynomial f. Given more than one function, $f_1, ..., f_p$, and an holonomic distribution u he obtains similarly functional equations of the type:

$$B_k(s_1, ..., s_p) f_1^{s_1} ... f_p^{s_p} u = P_k(x, \partial x, u) f_1^{s_1} ... f_k^{s_k+1} f_p^{s_p} u.$$

Here B_k is a polynomial in s, product of affine factors depending on a finite set $\mathcal{L}$ of linear parts $L(s_1, ..., s_k)$. This result allows him, for example, to prove that the integrals:

$$I(s, \sigma) = \int_{\mathbb{C}^n} |f|^{2s} |g|^{2\sigma} dx \wedge dx$$

can be extended as meromorphic functions with poles along lines with rational coefficients. When u generates a regular holonomic module, it can be proved (2^{nd} paper quoted above) that the set $\mathcal{L}$ of linear forms depends only on the characteristic variety of $\mathcal{D}u$. The proof of the first result is sketched at the end of the paper. It relies on the introduction of V- filtrations indexed by $\mathbb{Z}^p$ of $\mathcal{D}$-modules with support on the graph of $(f_1, ..., f_p)$, and on the Rees' rings and modules associated to these filtrations. The key point of this proof uses a theorem similar to the "flatenning theorem" of Hironaka proved by the author and F. Castro in the same papers.

Reviewer: J.M.Granger

MSC:

32A20 Meromorphic functions of several complex variables
32A25 Integral representations; canonical kernels (Szegő, Bergman, etc.)
32C30 Integration on analytic sets and spaces, currents

Keywords:

Bernstein-Sato polynomial; holonomic distribution

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