Arratia, Richard; **Liggett, Thomas M.**; **Williamson, Malcolm J.**

Scale-free and power law distributions via fixed points and convergence of (thinning and conditioning) transformations. (English) Zbl 1320.60010

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Summary: In discrete contexts such as the degree distribution for a graph, scale-free has traditionally been defined to be power-law. We propose a reasonable interpretation of scale-free, namely, invariance under the transformation of p-thinning, followed by conditioning on being positive.

For each $\beta \in (1, 2)$, we show that there is a unique distribution which is a fixed point of this transformation; the distribution is power-law-β, and different from the usual Yule-Simon power law-β that arises in preferential attachment models.

In addition to characterizing these fixed points, we prove convergence results for iterates of the transformation.

MSC:

60B10 Convergence of probability measures
05C82 Small world graphs, complex networks (graph-theoretic aspects)

Keywords:

thinning; power-law; scale-free; degree distribution; Pareto distribution

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