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Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates.

(English) Zbl 0638.30030
J. Lond. Math. Soc., II. Ser. 37, No. 1, 88-104 (1988).

Let f be a transcendental meromorphic function, and let k and j be integers that satisfy $k > j \geq 0$. In this paper we prove upper estimates for $|f^{(k)}(z)/f^{(j)}(z)|$ which hold for $z \in G$ where G is some specified set of points. For examples, we state the following two corollaries of our results.

Corollary 1: Let f have finite order ρ, and let $\epsilon > 0$ be a given constant. Then there exists a set $E \subset [0, 2\pi)$ that has linear measure zero, such that if $\psi_0 \in [0, 2\pi) - E$, then there is a constant $R_0 = R_0(\psi_0) > 0$ such that for all z satisfying arg z= ψ_0 and $|z| \geq R_0$, we have

$$|f^{(k)}(z)/f^{(j)}(z)| \leq |z|^{(k-j)(\rho-1+\epsilon)}.$$

Corollary 2: Let f have finite order ρ, and let $\epsilon > 0$ be a given constant. Then there exists a set $E \subset (1, \infty)$ that has finite logarithmic measure, such that for all z satisfying $|z| \notin [0, 1] \cup E$, we have

$$|f^{(k)}(z)|f^{(j)}(z)| \leq |z|^{(k-j)(\rho-1+\epsilon)}.$$

The gamma function gives examples which show that the estimates in Corollaries 1 and 2 are both sharp. We also prove an estimate (Corollary 3) like Corollary 2 in the case when the exceptional set E has finite linear measure, and this estimate turns out to be sharp by examples of Hayman. The four general theorems in the paper apply to infinite order functions as well, since these general theorems are stated in terms of the Nevanlinna characteristic and the counting functions of zeros and poles. One of the four theorems is sharp up to a constant factor. Estimates like those in this paper have countless applications in meromorphic function theory and in algebraic differential equations.

Reviewer: G.G.Gunderson

MSC:

30D35 Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

Keywords:

Nevanlinna characteristic

Full Text: DOI