Breuil, Christophe; Diamond, Fred

Hilbert modular forms modulo p and values of extensions between Galois characters. (Formes modulaires de Hilbert modulo p et valeurs d'extensions entre caractères galoisiens.)

(French. English summary) Zbl 1309.11046
Ann. Sci. Éc. Norm. Supér. (4) 47, No. 5, 905-974 (2014).

Let F be a totally real number field, v a place of F of characteristic p, and F_v the completion of F at v (supposed unramified over $\mathbb{Q}_p$): the aim of the paper under review is to gain a better understanding of certain smooth representations of $\mathrm{GL}_2(F_v)$ with values in $\mathrm{GL}_2(\overline{\mathbb{F}_p})$; as the situation is much better understood when $F_v = \mathbb{Q}_p$, the authors focus on the opposite case $F_v \neq \mathbb{Q}_p$.

In [the first author and *V. Paškūnas*, Mem. Am. Math. Soc. 1016, 114 p. (2012; Zbl 1245.22010)], families of smooth admissible representations of $\mathrm{GL}_2(F_v)$ have been constructed that depend on a very large number of parameters: in general, these parameters have not yet been given a satisfactory interpretation, but the work under consideration sheds some light on their meaning for a specific class of representations (those that are locally reducible and non-split).

More precisely, the representations considered here are of the following form: let $\overline{\rho} : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_2(\overline{\mathbb{F}_p})$ be a continuous, irreducible, totally odd representation, and suppose that its restriction to $\mathrm{Gal}(\overline{F_v}/F_v)$ is reducible and generic (in the sense of [loc. cit.]); assume furthermore that $\overline{\rho}$ is modular, i.e., it arises from the mod-p étale cohomology of a tower of Shimura curves. The first main result of the paper associates with $\overline{\rho}$ a collection of scalar invariants (denoted $x(J)$, where J is a family of embeddings of k_v in $\overline{\mathbb{F}_p}$) with values in $\overline{\mathbb{F}_p}^\times$; these $x(J)$ are a (usually small) subset of the parameters of [loc. cit.]. The second main theorem of the paper shows that the values of the $x(J)$'s can actually be computed explicitly, and only depend on the restriction of $\overline{\rho}$ to $\mathrm{Gal}(\overline{F_v}/F_v)$ (they are "local"): this is in stark contrast with the situation of [loc. cit.], where smooth admissible representations of $\mathrm{GL}_2(F_v)$ are constructed that have fixed $\mathrm{GL}_2(\mathcal{O}_{F_v})$-socle, but for which the invariants $x(J)$ can take essentially arbitrary values (the crucial difference here being of course the modularity assumption). This second result also implies that the $x(J)$'s encode the (non-split) extension between the two characters appearing in $\mathrm{Gal}\left(\overline{F}_v/F_v\right)$, thus giving a satisfactory interpretation of these parameters in the case of locally reducible representations.

Finally, an appendix is also included that contains the determination of the mod-p semisimplification of the Bushnell-Kutzko types of GL_2 and of the units in a quaternion algebra.

Reviewer: Davide Lombardo (Orsay)

MSC:

11F80 Galois representations
11F41 Automorphic forms on GL(2); Hilbert and Hilbert-Siegel modular groups and their modular and automorphic forms; Hilbert modular surfaces
11S37 Langlands-Weil conjectures, nonabelian class field theory
22E50 Representations of Lie and linear algebraic groups over local fields

Keywords:
Galois representations; Hilbert modular forms; local Langlands correspondence

Full Text: DOI Link