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The Jacobi matrix for functions in noncommutative algebras. (English) Zbl 1316.65049
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The authors develop a general tool for constructing the exact Jacobi matrix for functions defined in noncommutative algebraic systems without using any partial derivative. The construction is applied to solving nonlinear problems of the form $f(x) = 0$ with the aid of Newton's method in algebras defined in \mathbb{R}^N .

Reviewer: [Zhihua Zhang \(Beijing\)](#)

MSC:

65F30 Other matrix algorithms (MSC2010)

65F60 Numerical computation of matrix exponential and similar matrix functions

Cited in **1** Review
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Keywords:

Jacobi matrices in algebras over \mathbb{R}^N ; Jacobi matrices for polynomials over noncommutative algebras; Jacobi's matrix for the algebraic Riccati equation; Jacobi matrices for functions defined via Taylor and Laurent expansions over noncommutative algebras

Software:

[mftoolbox](#)

Full Text: [DOI](#)

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