Goryainov, V. V.
Evolution families of conformal mappings with fixed points and the Löwner-Kufarev equation. (English. Russian original) | Zbl 1321.30012 |
Sb. Math. 206, No. 1, 33-60 (2015); translation from Mat. Sb. 206, No. 1, 39-68 (2015).

The author develops an analogue of the Löwner-Pommerenke parametric method to study interior problems in the theory of univalent functions with constrains on the angular derivative at a boundary point. To do this, he utilizes the class $\mathcal{L}[0,1]$ of univalent conformal mappings f of the unit disc $\mathbb{D}$ onto itself ($f(0) = 0$, and f has a finite angular derivative at $z = 1$).

The main result reads:

Theorem. Let $\{w_{t,s} : 0 \leq s \leq t \leq T\}$ be an evolution family in $\mathcal{L}[0,1]$ and let $\beta(t) = w'_{t,0}(1)$ be an absolutely continuous function on $[0,T]$. Then for any $z \in \mathbb{D}$ and $s \in [0,T]$, the function $t \mapsto w_{t,s}(z)$ is absolutely continuous on $[s,T]$ and, for almost all t,

$$\frac{\partial}{\partial t} w_{t,s}(z) = -\frac{\beta'(t)}{\beta(t)} w_{t,s}(z)(1 - w_{t,s}(z))H(w_{t,s}(z),t),$$

where $H(z,t)$ is a function defined on $\mathbb{D} \times [0,T]$, z-holomorphic and t-measurable, and for almost all $t \in [0,T]$ the function $H(\cdot,t)$ is represented in the form

$$H(z,t) = \int_{\mathbb{T}} \frac{1 - \chi}{1 - \chi z} d\mu(\chi)$$

with some probability measure μ on $\mathbb{T}$.

Under the conditions of this theorem, by changing the time scale, a normalized evolution family with $\beta(t) = e^t$, $0 \leq t \leq T$, is obtained.

It is also shown that each mapping f in the semigroup $\mathcal{L}[0,1]$ can be embedded into an evolution family that is generated by some infinitesimal generating function H.

Reviewer: Sergei V. Rogosin (Minsk)

MSC:

30C55	General theory of univalent and multivalent functions of one complex variable
30D05	Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable
30C35	General theory of conformal mappings
30C75	Extremal problems for conformal and quasiconformal mappings, other methods

Cited in **6** Documents

Keywords:

conformal mapping; univalent functions; Löwner-Kufarev equation; fixed point; evolution family; angular derivative

Full Text: DOI