Gresho, Philip M.; **Sani, Robert L.**

On pressure boundary conditions for the incompressible Navier-Stokes equations. (English)

Zbl 0644.76025
Int. J. Numer. Methods Fluids 7, 1111-1145 (1987).

The authors have very successfully analysed various situations where the boundary conditions for the pressure Poisson equation would lead to discrepancies. They have attempted to settle the issue (1) via the continuum partial differential equations, (2) via the analysis of several consistent discretized approximations to the PDEs and (3) by numerical examples.

The following observations are noted. (i) To solve the continuum Poisson equation for the pressure only the Neumann boundary condition is always appropriate; i.e., it provides a unique solution to $t \geq 0$. The Dirichlet boundary condition is generally only appropriate for $t > 0$; it often does not apply at $t = 0$. The unique solution obtained using either boundary condition will, for $t > 0$, satisfy the often boundary condition provided the Neumann boundary condition is applied at $t = 0$. (ii) Any consistent discrete approximation of the original (primitive) equations contains, as an automatic built-in boundary condition for the (implied) discrete pressure poisson equation, the Neumann boundary condition; for $t \geq 0$. It does not obviously satisfy the Dirichlet boundary condition. (iii) The converged numerical solution from (ii) will, however, also satisfy the Dirichlet boundary condition owing to (i); but in general for $t > 0$. These observations have proper basis for all numerical approximation methods and the authors hope to extend this work in other directions where computational difficulties often occur.

Reviewer: V.Subba Rao

MSC:

76D05	Navier-Stokes equations for incompressible viscous fluids	Cited in **1** Review
35Q99	Partial differential equations of mathematical physics and other areas of application	Cited in **146** Documents

Keywords:

boundary conditions; Poisson equation; discretized approximations; Neumann boundary condition; Dirichlet boundary condition; unique solution

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References:

[1] and , 'Numerical simulation of viscous incompressible flows', in Ann. Rev. Fluid Mech., Annual Reviews, Palo Alto, CA, 1974, pp. 281-319.

[2] and , Numerical Analysis of Spectral Methods: Theory and Applications, SIAM, Philadelphia, PA, 1977.· doi:10.1137/1.9781611970425

[3] Moin, J. Comput. Phys. 3 pp 381– (1980)

[4] Dierieck, Int. j. numer. fluids 7 pp 69– (1987)

[5] Donea, Int. j. numer. methods eng. 17 pp 790– (1981)

[6] Gresho, Int. j. numer. methods eng. 18 pp 1260– (1982)

[7] and , 'On the divergence-free (i.e., mass conservation, solenoidal) condition in computational fluid dynamics: How important is it?', in et al. (eds), Numerical Methods in Laminar and Turbulent Flow, Pineridge Press, Swansea, UK, 1983. · Zbl 0551.76026

[8] Gustafson, J. Comput. Phys. 64 pp 279– (1986)

[9] Deville, Int. j. numer. methods fluids 4 pp 1149– (1984)

[10] Heywood, SIAM J. Numer. Anal. 19 pp 275– (1982)

[11] Temam, J. Differential Equations 43 pp 73– (1982)

[12] Morino, Comput. Mech. 1 pp 65– (1986)

[13] Strikwerda, SIAM J. Sci. Stat. Comput. 5 pp 56– (1984)

[14] 'Comments on finite difference methods for the Stokes and Navier-Stokes equations', Int. j. numer. methods fluids, submitted

to (1987).

[15] Navier-Stokes Equations, 3rd Edn, North-Holland, Amsterdam and New York, 1985.

[16] Heywood, Indiana Univ. Math. J. 29 pp 639– (1980)

[17] Chorin, Math. Comput. 22 pp 745– (1968)

[18] and , 'On the time-dependent solution of the incompressible Navier-Stokes equations in two and three dimensions', in Recent Advances in Numerical Methods in Fluids, Vol. 1, Pineridge Press, Swansea, UK, 1980.

[19] and , 'Numerical solution of nonlinear differential/algebraci systems from physics and engineering', in et al. (eds), Proc. Int. Conf. on Innovative Methods for Nonlinear Problems, Pineridge Press, Swansea, UK, 1984.

[20] Briley, J. Comput. Phys. 14 pp 8– (1974)

[21] and , 'Study of incompressible Navier-Stokes equations in primitive variables using implicit numerical technique', Proc. AIAA 3rd Computational Fluid Dynamics Conf., Albuquerque, June 1977, p. 156; also AAIA Paper 77-648.

[22] Ghia, AIAA J. 17 pp 298– (1979)

[23] 'On the Neumann problem for the pressure in a Navier-STokes model', in Proc. 2nd Int. Conf. on Numerical Methods in Laminar and Turbulent Flow, Pineridge Press, Swansea, UK, 1981, p. 389.

[24] Introduction to Applied Mathematics, Wellesley-Cambridge Press, Wellesley, MA, 1986. · Zbl 0618.00015

[25] Sani, Int. j. numer. methods fluids 1 pp 17– (1981)

[26] Engelmen, Int. j. numer. methods fluids 2 pp 25– (1982)

[27] Chorin, Stud. Numer. Anal. 2 pp 64– (1968)

[28] Engelman, Int. j. numer. methods fluids 2 pp 225– (1982)

[29] Numerical Methods for Nonlinear Variational Problems, Springer, New York, Appendix III, 1984.· doi:10.1007/978-3-662-12613-4

[30] Gresho, Int. j. numer. methods fluids 4 pp 557– (1984)

[31] and , 'A new semi-implicit method for solving the time-dependent conservation equations for incompressible flow', in et al. (eds), Numerical Methods in Laminar and Turbulent Flow, Part 1, Pineridge Press, Swansea, UK, 1985 (also available as UCRL-92505, LLNL).

[32] Harlow, Phys. Fluids 8 pp 2181– (1965)

[33] Williams, J. Fluid Mech. 37 pp 727– (1969)

[34] Computational Fluid Dynamics, Hermosa Press, Albuquerque, NM, 1976.

[35] and , Computational Methods for Fluid Flow, Springer, New York, 1983. · Zbl 0514.76001 · doi:10.1007/978-3-642-85952-6

[36] Incompressible Flow, Wiley, New York, 1984, p. 336.

[37] Gresho, Computers and Fluids 9 pp 223– (1981)

[38] Gresho, Numer. Heat Transfer 2 pp 519– (1979)

[39] and , Conduction of Heat in Solids, 2nd Edn, Clarendon Press, Oxford, 1959.

[40] and , 'A Stokes flow test problem and some isoparametric finite element results', submitted to Commun, Appl. Numer. Methods (1987).

[41] Application of Green's Functions in Science and Engineering, Prentice-Hall, Englewood Cliffs, NJ, 1971.

[42] Green's Functions and Boundary Value Problems, Wiley, New York, 1979.

[43] Glowinski, Numer. Math. 33 pp 424– (1979)

[44] Implementation of Finite Element Methods for Navier-Stokes Equations, Springer, New York, 1981. · Zbl 0475.76036 · doi:10.1007/978-3-642-87047-7

[45] and , 'Treatment of incompressibility and boundary conditions in 3-D numerical spectral simulations of plane channel flows', in (ed.), Notes on Numerical Fluid Mechanics, Vol. 2, Vieweg, Braunschweig, 1980, p. 165. · Zbl 0463.76020

[46] Marcus, J. Fluid Mech. 146 pp 45– (1984)

[47] le Quere, J. Comput. Phys. 57 pp 210– (1985)

[48] Quartapelle, J. Comput. Phys. 62 pp 340– (1986)

[49] and , A Mathematical Introduction to Fluid Mechanics, Springer, New York, 1979. · Zbl 0417.76002 · doi:10.1007/978-1-4684-0082-3

[50] Pironneau, C. R. Acad. Sci. Paris, Ser. 1 303 pp 403– (1986)

[51] Pironneau, C. R. Acad. Sci. Paris. Ser. I 304 pp 23– (1987)

[52] 'Development and application of high-order numerical methods for solution of the three-dimensional Navier-Stokes equations', Ph.D. Thesis, Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA (1986).