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Convergence analysis of the summation of the factorially divergent Euler series by Padé approximants and the delta transformation. (English) Zbl 1325.65008

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Summary: Sequence transformations are valuable numerical tools that have been used with considerable success for the acceleration of convergence and the summation of diverging series. However, our understanding of their theoretical properties is far from satisfactory. The Euler series $\mathcal{E}(z) \sim \sum_{n=0}^{\infty}(-1)^n n! z^n$ is a very important model for the ubiquitous factorially divergent perturbation expansions in theoretical physics and for the divergent asymptotic expansions for special functions. In this article, we analyze the summation of the Euler series by Padé approximants and by the delta transformation, which is a powerful nonlinear Levin-type transformation that works very well in the case of strictly alternating convergent or divergent series. Our analysis is based on a very recent factorial series representation of the truncation error of the Euler series. We derive explicit expressions for the transformation errors of Padé approximants and of the delta transformation. A subsequent asymptotic analysis proves *rigorously* the convergence of both Padé and delta. Our asymptotic estimates clearly show the superiority of the delta transformation over Padé. This is in agreement with previous numerical results.

MSC:

65B10 Numerical summation of series
41A21 Padé approximation
40D05 General theorems on summability
40G99 Special methods of summability

Keywords:

Euler series; summation of factorial divergence; Weniger's delta transformation; Padé approximants; convergence proofs

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