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A counterexample of approximating an almost isometric operator by an isometric operator in $B(\ell^1 \to \ell^\infty)$ space. (Chinese. English summary) [Zbl 0645.41017]


Let $E_i (i = 1, 2)$ be a Banach space, $T \in B(E_1 \to E_2)$. If there is $\epsilon > 0$ such that $(1 + \epsilon)\|X\| \leq \|TX\| \leq (1 + \epsilon)\|X\| (\forall X \in E_1)$, then $T$ is called $\epsilon$-isometric operator. All $\epsilon$-isometric operators are called almost isometric operators. In 1966, M. Michael and A. Pelewzynski [Isr. J. Math. 4, 189-198 (1966; Zbl 0151.176]) proved that for an arbitrary $\epsilon$-isometric operator $T \in B(\ell^\infty_n \to \ell^\infty_m)$ always there exists an isometric operator such that $\|T - s\| \leq \epsilon$.

Then, for an arbitrary space $B(E_1 \to E_2)$ and an arbitrary $\epsilon$-isometric operator $T \in B(E_1 \to E_2)$, is there an isometric operator $S \in B(E_1 \to E_2)$ such that $\|T - s\| \leq \delta(\epsilon)$ ($\delta (\epsilon) \to 0, \epsilon \to 0$)? In this note, a counterexample is constructed to illustrate the impossibility of approximating an almost isometric operator by an isometric operator in the $B(R' \to R^\infty)$ space.

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