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On the convergence of the conditional gradient method as applied to the optimization of an elliptic equation. (English. Russian original) Zbl 1318.49055

Comput. Math. Math. Phys. 55, No. 2, 212-226 (2015); translation from Zh. Vychisl. Mat. Mat. Fiz. 55, No. 2, 213-228 (2015).

Summary: The optimal control of a second-order semilinear elliptic diffusion-reaction equation is considered. Sufficient conditions for the convergence of the conditional gradient method are obtained without using assumptions (traditional for optimization theory) that ensure the Lipschitz continuity of the objective functional derivative. The total (over the entire set of admissible controls) preservation of solvability, a pointwise estimate for solutions, and the uniqueness of a solution to the homogeneous Dirichlet problem for a controlled elliptic equation are proved as preliminary results, which are of interest on their own.

MSC:

49M30 Other numerical methods in calculus of variations (MSC2010)
49J20 Existence theories for optimal control problems involving partial differential equations
35J61 Semilinear elliptic equations

Keywords:

optimal control; semilinear elliptic diffusion-reaction equations; conditional gradient method; total preservation of solvability; solution estimate; solution uniqueness

Full Text: DOI

References:

[1] F. P. Vasil'ev, \textit{Optimization Methods} (Faktorial, Moscow, 2002) [in Russian].

[2] Arguchintsev, A V; Dykhta, V A; Srochko, V A, Optimal control: nonlocal conditions, computational methods, and the variational principle of maximum, Russ. Math., 53, 1-35, (2009) · Zbl 1183.49003

[3] R. P. Fedorenko, \textit{Approximate Solution of Optimal Control Problems} (Nauka, Moscow, 1978) [in Russian]. · Zbl 0462.49001

[4] Vorontsov, M A; Zheleznykh, N I; Potapov, M M, A gradient procedure for intra-resonator control of light beams, USSR Comput. Math. Math. Phys., 30, 74-79, (1990)

[5] Srochko, V A, Modernization of gradient-type methods in optimal control problems, Russ. Math., 46, 64-76, (2002) · Zbl 1063.49023

[6] Golichev, I I; Lookmanov, R L, A method for the expansion of a function of an operator in some problems of optimal control, Math. Notes, 47, 322-328, (1990) · Zbl 0718.49023

[7] Kabanikhin, S I; Iskakov, K T, Justification of the steepest descent method for the integral statement of an inverse problem for a hyperbolic equation, Sib. Math. J., 42, 478-494, (2001) · Zbl 0981.35097

[8] Sumin, M I, Parametric dual regularization for an optimal control problem with pointwise state constraints, Comput. Math. Math. Phys., 49, 1987-2005, (2009)

[9] L. V. Kantorovich and G. P. Akilov, \textit{Functional Analysis} (Pergamon, Oxford, 1982; Nauka, Moscow, 1984). · Zbl 0484.46003

[10] M. Minoux, \textit{Mathematical Programming: Theory and Algorithms} (Wiley, New York, 1986). · Zbl 0602.90090

[11] F. Tröltzsch, \textit{Optimal Control of Partial Differential Equations: Theory, Methods, and Applications} (Am. Math. Soc., Providence, RI, 2010).

[12] A. Kh. Vorob'ev, \textit{Diffusion Problems in Chemical Kinetics} (Mosk. Gos. Univ., Moscow, 2003) [in Russian].

[13] Lubyshev, F V; Manapova, A R, Difference approximations of optimization problems for semilinear elliptic equations in a convex domain with controls in the coefficients multiplying the highest derivatives, Comput. Math. Math. Phys., 53, 8-33, (2013) · Zbl 1274.49007

[14] Vakhitov, I S, Inverse identification problem for unknown coefficient in the diffusion-reaction equation, Dal'nevost. Mat. Zh., 10, 93-105, (2010)

[15] Chernov, A V, On the convergence of the conditional gradient method in distributed optimization problems, Comput. Math. Math. Phys., 51, 1510-1523, (2011) · Zbl 1274.49037

[16] Sumin, V I, The features of gradient methods for distributed optimal-control problems, USSR Comput. Math. Math. Phys., 30, 1-15, (1990) · Zbl 0719.49003

[17] O. A. Ladyzhenskaya and N. N. Uraltseva, \textit{Linear and Quasilinear Elliptic Equations} (Nauka, Moscow, 1973; Academic, New York, 1987). · Zbl 0269.35029

[18] D. Gilbarg and N. S. Trudinger, \textit{Elliptic Partial Differential Equations of Second Order} (Springer-Verlag, Berlin, 1983; Nauka, Moscow, 1989). · Zbl 0562.35001

[19] Potapov, D K, Control problems for equations with a spectral parameter and a discontinuous operator under perturbations, Zh. Sib. Fed. Univ. Ser. Mat., 5, 239-245, (2012)

[20] Chernov, A V, On a generalization of the method of monotone operators, Differ. Equations, 49, 517-527, (2013) · Zbl 1279.47079

[21] Chernov, A V, A majorant criterion for the total preservation of global solvability of controlled functional operator equation, Russ. Math., 55, 85-95, (2011) · Zbl 1244.47064

[22] Chernov, A V, A majorant-minorant criterion for the total preservation of global solvability of a functional operator equation, Russ. Math., 56, 55-65, (2012) · Zbl 1345.39013

[23] M. M. Karchevskii and M. F. Pavlova, \textit{Equations of Mathematical Physics: Additional Chapters} (Kazan. Gos. Univ., Kazan, 2012) [in Russian].

[24] M. A. Krasnosel'skii, \textit{Topological Methods in the Theory of Nonlinear Integral Equations} (Gostekhteorizdat, Moscow, 1956; Pergamon, New York, 1964). · Zbl 0070.33001