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Generalized contractive multivalued mappings and their fixed points. (English)

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Let us denote by (X,d) a metric space, by $P_x(X)$ the family of all bounded proximinal subsets of X and by H the Hausdorff metric induced by d on $P_x(X)$. Let T be a multivalued mapping, $T : X \to P_x(X)$, such that $H(Tx, Ty) \leq h(x, y)d(x, y), \forall x, y \in X$, where h satisfies $\sup\{h(x, y) : a \leq d(x, y) \leq b\} < 1$ for each finite closed interval $[a, b] \in (0, +\infty)$. Moreover assume that if $(x_n, y_n) \in X \times X$ and $d(x_n, y_n) \to 0$, then $h(x_n, y_n) \to k$ for some $k \in [0, 1]$. Then T has a fixed point in X. These hypotheses are obviously satisfied if $h(x, y) = \alpha(d(x, y))$, where α is a monotone increasing function such that $0 \leq \alpha(t) < 1$ for each $t \in (0, +\infty)$. The result is connected with some known theorems in the setting of point-to-point mappings.

Reviewer: D.Roux

MSC:

54H25 Fixed-point and coincidence theorems (topological aspects)

Keywords:

contractive multivalued map; proximinal sets