Linde, J.; de la Puente, M. J.
Matrices commuting with a given normal tropical matrix. (English) Zbl 1321.15046
Linear Algebra Appl. 482, 101-121 (2015).

Summary: Consider the space M_n^{nor} of square normal matrices $X = (x_{ij})$ over $\mathbb{R} \cup \{-\infty\}$, i.e., $-\infty \leq x_{ij} \leq 0$ and $x_{ii} = 0$. Endow M_n^{nor} with the tropical sum $\oplus$ and multiplication $\odot$. Fix a real matrix $A \in M_n^{\text{nor}}$ and consider the set $\Omega(A)$ of matrices in M_n^{nor} which commute with A. We prove that $\Omega(A)$ is a finite union of alcoved polytopes; in particular, $\Omega(A)$ is a finite union of convex sets. The set $\Omega^A(A)$ of X such that $A \odot X = X \odot A = A$ is also a finite union of alcoved polytopes. The same is true for the set $\Omega'(A)$ of X such that $A \odot X = X \odot A = X$.

A topology is given to M_n^{nor}. Then, the set $\Omega^A(A)$ is a neighborhood of the identity matrix I. If A is strictly normal, then $\Omega'(A)$ is a neighborhood of the zero matrix. In one case, $\Omega(A)$ is a neighborhood of A. We give an upper bound for the dimension of $\Omega'(A)$. We explore the relationship between the polyhedral complexes $\operatorname{span} A$, $\operatorname{span} X$ and $\operatorname{span}(AX)$, when A and X commute. Two matrices, denoted $\underline{A}$ and $\overline{A}$, arise from A, in connection with $\Omega(A)$. The geometric meaning of them is given in detail, for one example. We produce examples of matrices which commute, in any dimension.

MSC:

15A80 Max-plus and related algebras
14T05 Tropical geometry (MSC2010)
15B57 Hermitian, skew-Hermitian, and related matrices

Keywords:

tropical algebra; commuting matrices; normal matrix; idempotent matrix; alcoved polytope; convexity

Software:

TropLi

Full Text: DOI arXiv

References:

[1] Akian, M.; Bapat, R.; Gaubert, S., MAX-plus algebra, (Hobgen, L., Handbook of Linear Algebra, (2007), Chapman and Hall), Chapter 25

[2] Baccelli, F. L.; Cohen, G.; Olsder, G. J.; Quadrat, J. P., Syncronization and linearity, (1992), John Wiley Chichester, New York

[3] Brugallé, E., Un peu de géométrie tropicale, Quadrature, 74, 10-22, (2009) · Zbl 1202.14055

[4] Brugallé, E., Some aspects of tropical geometry, Eur. Math. Soc. Newsl., 83, 23-28, (2012) · Zbl 1285.14069

[5] Butkovič, P., MAX-algebra: the linear algebra of combinatorics?, Linear Algebra Appl., 367, 313-335, (2003) · Zbl 1022.15017

[6] Butkovič, P., Simple image set of $(\max, +)$ linear mappings, Discrete Appl. Math., 105, 73-86, (2000) · Zbl 0976.15013

[7] Butkovič, P., MAX-plus linear systems: theory and algorithms, (2010), Springer

[8] Cuninghame-Green, R., Minimax algebra, Lecture Notes in Econom. and Math. Systems, vol. 166, (1970), Springer · Zbl 0498.90084

[9] Cuninghame-Green, R. A., Minimax algebra and applications, (Hawkes, P., Adv. Imaging Electron Phys., vol. 90, (1995), Academic Press), 1-121

[10] Develin, M.; Sturmfels, B.; Develin, M.; Sturmfels, B., Tropical convexity, Doc. Math., Doc. Math., 9, 205-206, (2004), (Erratum) (electronic) · Zbl 1054.52004

[11] Gathmann, A., Tropical algebraic geometry, Jahresber. Dtsch. Math.-Ver., 108, 1, 3-32, (2006) · Zbl 1109.14038

[12] Itenberg, I.; Brugallé, E.; Tessier, B., Géométrie tropicale, (2008), Editions de l'École Polythecnique

[13] Itenberg, I.; Mikhalkin, G.; Shustin, E., Tropical algebraic geometry, (2007), Birkhäuser · Zbl 1162.14300

[14] Izhakian, Z.; Johnson, M.; Kambites, M., Pure dimension and projectivity of tropical politopes, (2012)

[15] Izhakian, Z.; Johnson, M.; Kambites, M., Tropical matrix groups, (2012)

[16] Jiménez, A.; de la Puente, M. J., Characterizing the convexity of the \textit{n}-dimensional tropical simplex and the six maximal convex classes in $\mathbb{R}^3$, (2012)

[17] Johnson, M.; Kambites, M., Idempotent tropical matrices and finite metric spaces, Adv. Geom., 14, 2, 253-276, (2014) · Zbl 1310.14050

[18] Joswig, M.; Kulas, K., Tropical and ordinary convexity combined, Adv. Geom., 10, 333-352, (2010) · Zbl 1198.14060

[19] Joswig, M.; Sturmfels, B.; Yu, J., Affine buildings and tropical convexity, Albanian J. Math., 1, 4, 187-211, (2007) · Zbl 1133.52003

[20] Katz, R.; Schneider, H.; Sergeev, S., On commuting matrices in MAX algebra and in classical nonnegative algebra, Linear Algebra Appl., 436, 276-292, (2012) · Zbl 1245.15026

[21] Kuhn, H. W., The Hungarian method for the assignment problem, Naval Res. Logist., 2, 83-97, (1955) · Zbl 0143.41905

[22] Lam, T.; Postnikov, A., Alcoved polytopes I, Discrete Comput. Geom., 38, 3, 453-478, (2007) · Zbl 1134.52019

[23] Lam, T.; Postnikov, A., Alcoved polytopes II, (2012) · Zbl 1414.52007

[24] (Litvinov, G. L.; Maslov, V. P., Idempotent Mathematics and Mathematical Physics, Proceedings Vienna 2003, Contemp. Math., vol. 377, (2005), American Mathematical Society) · Zbl 1069.00011

[25] Mikhalkin, G., What is a tropical curve?, Notices Amer. Math. Soc., 511-513, (2007) · Zbl 1142.14300

[26] Papadimitriou, C. H.; Steiglitz, K., Combinatorial optimization: algorithms and complexity, (1998), Prentice Hall, and corrected unabridged republication by Dover, 1982 · Zbl 0944.90066

[27] Prasolov, V. V., Problems and theorems in linear algebra, (1994), AMS · Zbl 0803.15001

[28] de la Puente, M. J., Tropical linear maps on the plane, Linear Algebra Appl., 435, 7, 1681-1710, (2011) · Zbl 1221.15008

[29] de la Puente, M. J., On tropical Kleene star matrices and alcoved polytopes, Kybernetika, 49, 6, 897-910, (2013) · Zbl 1297.15029

[30] J. Richter-Gebert, B. Sturmfels, T. Theobald, First steps in tropical geometry, in [24], pp. 289-317. · Zbl 1093.14080

[31] Rincón, F., Local tropical linear spaces, Discrete Comput. Geom., 50, 700-713, (2013) · Zbl 1281.14049

[32] Sergeev, S., MAX-plus definite matrix closures and their eigenspaces, Linear Algebra Appl., 421, 182-201, (2007) · Zbl 1131.15009

[33] Sergeev, S.; Scheneider, H.; Butkovič, P., On visualization, subeigenvectors and Kleene stars in MAX algebra, Linear Algebra Appl., 431, 2395-2406, (2009) · Zbl 1180.15027

[34] Speyer, D.; Sturmfels, B., Tropical mathematics, Math. Mag., 82, 3, 163-173, (2009) · Zbl 1227.14051

[35] Wagneur, E., Moduloïds and pseudomodules. dimension theory, Discrete Math., 98, 57-73, (1991) · Zbl 0757.06008

[36] Werner, A.; Yu, J., Symmetric alcoved polytopes, (2012) · Zbl 1302.52014

[37] Yoeli, M., A note on a generalization of Boolean matrix theory, Amer. Math. Monthly, 68, 6, 552-557, (1961) · Zbl 0115.02103

[38] K. Zimmermann, Extremální algebra, Výzkumná publikace ekonomicko-matematické laboratoře při ekonomickém ústavé ČSAV, vol. 46, Prague, 1976 (in Czech).