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Relation algebras and Schröder categories. (English) Zbl 0649.03047

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This paper begins with the definition of relation algebras $(A,+,\cdot,\ \ -,0,1,;,^{\sqcup},')$ and a recall of some of their properties. A Schröder category, as defined by Olivier and Serrato, is a category $\mathfrak{C}$ such that for every i,j$\in Ob\mathfrak{C}$, the set $\mathfrak{C}(i,j)$ is a Boolean algebra and there is an involution $^{\sqcup}:\mathfrak{C}(i,j)\to\mathfrak{C}(j,i)$; besides, the composition of morphism and $^{\sqcup}$ satisfies certain supplementary conditions. The author proves that if I is any finite set of objects of a Schröder category $\mathfrak{C}$, then the direct product of the Boolean algebras $\mathfrak{C}(i,j)$, $i,j\in I$, can be made into a relation algebra. Then, a natural concept of tensor product of Boolean algebras is introduced which turns out to be the same as the free product. The next section deals with Boolean modules. Further, given a system of simple relation algebras $\mathfrak{A}_i$, $i\in I$, the author constructs from them in a canonical way a Schröder category $\mathfrak{C}$ whose objects are the members of I in such a way that $\mathfrak{C}(i,i)=\mathfrak{A}_i$, and assuming that I is finite, the semi-product of the algebras $\mathfrak{A}_i$ is defined as a certain algebra $\mathfrak{A}$ of matrices a with entries $a(i,j)\in\mathfrak{C}(i,j)$; the algebras $\mathfrak{A}_i$ determine $\mathfrak{A}$ up to isomorphism. The last sections study in some detail the equivalence elements of a relation algebra (i.e., the elements u such that $u;u\leq u$ and $u^{\sqcup}=u$) and the relation algebras that are generated by an equivalence element. Every such algebra is finite and representable.

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MSC:

03G15	Cylindric and polyadic algebras; relation algebras
18D99	Categorical structures
06E99	Boolean algebras (Boolean rings)
08B25	Products, amalgamated products, and other kinds of limits and colimits

Keywords:

relation algebras; Schröder category; Boolean algebras; tensor product; free product; Boolean modules; equivalence element

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