Gouveia, João; Robinson, Richard Z.; Thomas, Rekha R.
Worst-case results for positive semidefinite rank. (English) Zbl 1344.90046
Math. Program. 153, No. 1 (B), 201-212 (2015).

The positive semidefinite rank is an extension of the notion of the nonnegative rank of a matrix. The nonnegative rank of a matrix is the smallest possible integer k such that each entry (i,j) of the matrix can be written as the inner product of two vectors a_i and b_j in the cone of nonnegative vectors of size k. The positive semidefinite rank is the smallest possible integer k such that each entry (i,j) of the matrix can be written as the inner product of two matrices A_i and B_j in the cone of positive semidefinite matrices of size k. These ranks can be interpreted as a measure of complexity of a polytope, in the sense that they encode the size of a sparse representation of a polytope through a factorization of its matrix description. The paper under review reports on state-of-the-art results on lower bounds on the positive semidefinite rank, both for generic polytopes and for specific polytopes.

Reviewer: Didier Henrion (Toulouse)

MSC:

90C22 Semidefinite programming
52B11 n-dimensional polytopes
15A23 Factorization of matrices

Keywords:
representations of polytopes; matrix factorization; semidefinite programming

Full Text: DOI arXiv

References:

[1] Bochnak, J., Coste, M., Roy, M.: Real Algebraic Geometry. Springer, Berlin (1998) · Zbl 0912.14023
[2] Braun, G., Fiorini, S., Pokutta, S., Steurer, D.: Approximation limits of linear programs (beyond hierarchies). In: Proceedings of FOCS (2012) · Zbl 1343.68308
[3] Briët, J., Dadush, D., Pokutta, S.: On the existence of $0/1$ polytopes with high semidefinite extension complexity. Math. Program. Ser. B (to appear) · Zbl 1395.90241
[4] Cohen, JE; Rothblum, UG, Nonnegative ranks, decompositions, and factorizations of nonnegative matrices, Linear Algebra Its Appl., 190, 149-168, (1993) · Zbl 0784.15001
[5] Fawzi, H., Parrilo, P.A.: Lower bounds to nonnegative rank via nonnegative nuclear norms. Math. Progam. Ser. B (to appear) · Zbl 1327.90202
[6] Fiorini, S; Kaibel, V; Pashkovich, K; Theis, DO, Combinatorial bounds on nonnegative rank and extended formulations, Discret. Math., 313, 67-83, (2013) · Zbl 1259.90111
[7] Fiorini, S., Massar, S., Pokutta, S., Tiwary, H.R., de Wolf, R.: Linear vs. semidefinite extended formulations: exponential separation and strong lower bounds. In: Proceedings of STOC (2012) · Zbl 1286.90125
[8] Fiorini, S; Rothvoss, T; Tiwary, H, Extended formulations for polygons, Discret. Comput. Geom., 48, 658-668, (2012) · Zbl 1290.68122
[9] Gillis, N; Glineur, F, On the geometric interpretation of the nonnegative rank, Linear Algebra Its Appl., 437, 2685-2712, (2012) · Zbl 1258.65039
[10] Gouveia, J; Parrilo, PA; Thomas, RR, Lifts of convex sets and cone factorizations, Math. Oper. Res., 38, 248-264, (2013) · Zbl 1291.90172
[11] Gouveia, J; Robinson, RZ; Thomas, RR, Polytopes of minimum positive semidefinite rank, Discret. Comput. Geom., 50, 679-699, (2013) · Zbl 1279.52023
[12] Hungerford,T.: Algebra. Holt, Rinehart, and Winston, New York (1974)
[13] Lee T., Theis, D.O.: Support based lower bounds for the positive semidefinite rank of a nonnegative matrix. Linear Algebra Appl. (to appear) · Zbl 1290.68122
[14] Moitra, A.: An almost optimal algorithm for computing nonnegative rank. In: Proceedings of the SODA (2013) · Zbl 1334.68102
[15] Pashkovich, K.: Extended Formulations for Combinatorial Polytopes. PhD thesis, Magdeburg Universität (2012) · Zbl

1259.90111

[16] Renegar, J, On the computational complexity and geometry of the first-order theory of the reals, J. Symb. Comp., 13, 255-352, (1992) · Zbl 0763.68042

[17] Shitov, Y, An upper bound for nonnegative rank, J. Comb. Theory Ser. A, 122, 126-132, (2014) · Zbl 1316.52020

[18] Vavasis, S, On the complexity of nonnegative matrix factorization, SIAM J. Optim., 20, 1364-1377, (2009) · Zbl 1206.65130

[19] Wolkowicz, H., Saigal, R., Vandenberghe, L.: Handbook of Semidefinite Programming. Kluwer, Boston (2000) · Zbl 0962.90001

[20] Yannakakis, M, Expressing combinatorial optimization problems by linear programs, J. Comput. Syst. Sci., 43, 441-466, (1991) · Zbl 0748.90074