Colmez, Pierre

Residue at $s=1$ of p-adic zeta functions. (Résidu en $s=1$ des fonctions zêta p-adiques.)

(French) Zbl 0651.12010

Invent. Math. 91, No. 2, 371-389 (1988).

Let F be a totally real degree n extension of $\mathbb{Q}$ and let O_F be its ring of integers. Let p be a prime integer. The p-adic zeta function $\zeta_{F,\mathfrak{p}}$ was defined in different ways by Serre, Deligne and Ribet, Pierrette Cassous-Nogues and Barsky. Here the author shows that

$$\lim_{s\to 1}(s-1)\zeta_{F,\mathfrak{p}}(s)=2^n R_p h E_p(1)/w\sqrt{D} \tag{1}$$

where R_p is the p-adic regulator, h is the class number of F, $E_p(1)=\prod_{\mathfrak{p}|p,\mathfrak{p}\in O_F}(1-1/N(\mathfrak{p}))$ (with N the norm of F over $\mathbb{Q}$, which is not specified in the article), w is the number of roots of unity and D is the discriminant of F. (The equation (1) was already proven by *A. Amice* and *J. Fresnel* when F is an abelian extension in [Acta Arith. 20, 353–384 (1972; Zbl 0217.04303)]). The p-adic distribution introduced by Yvette Amice helps the author to translate P. Cassou-Nogues' results and deepen all of them in order to obtain inequalities about the Gauss norm of certain polynomials. He then computes the residue thanks to considerations on finitely generated subgroups in $\mathbb{R}^n$.

Reviewer: Alain Escassut (Aubière)

MSC:

11S40	Zeta functions and L-functions
11M38	Zeta and L-functions in characteristic p
11R29	Class numbers, class groups, discriminants
11R80	Totally real fields

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p-adic zeta function; p-adic regulator

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