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Reconstruction of polygonal shapes from sparse Fourier samples. (English) Zbl 1362.94012
J. Comput. Appl. Math. 297, 117-131 (2016).

In this interesting paper, the authors reconstruct the characteristic function $f(x_1, x_2) = 1_D(x_1, x_2)$ of a simply-connected polygonal domain $D \subset \mathbb{R}^2$ from relatively few samples of the Fourier transform \hat{f} . This reconstruction method is based on a stable Prony method (such as approximate Prony method, MUSIC or ESPRIT) for the recovery of univariate exponential sums. By this approach, the authors reconstruct the vertices of the polygon in a correct way. It is remarkable that this method works also for a non-convex polygonal domain D .

Note that the reconstruction of a convex polygonal domain $D \subset \mathbb{C}$ from given moments were presented by *G. H. Golub* et al. [*SIAM J. Sci. Comput.* 21, No. 4, 1222–1243 (1999; [Zbl 0956.65030](#))].

Reviewer: [Manfred Tasche](#) (Rostock)

MSC:

- [94A12](#) Signal theory (characterization, reconstruction, filtering, etc.)
- [42B10](#) Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
- [65D20](#) Computation of special functions and constants, construction of tables

Cited in **3** Documents

Keywords:

polygonal domain; polygonal shape reconstruction; non-convex polygonal domain; sparse Fourier reconstruction; Prony method

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