Namioka, I.
Radon-Nikodým compact spaces and fragmentability. (English) Zbl 0654.46017

As the author asserts in the introduction the paper gives “a relatively self-contained introduction” to RN compact spaces (i.e. topological spaces homeomorphic to weak*-compact subsets of dual Banach spaces with the RNP) and “develops characterizations of RN compact spaces” (free of Banach space context). The whole paper is based on the notion of fragmentability defined by J. E. Jayne and C. A. Rogers [Acta Math. 155, 41-79 (1985; Zbl 0588.54020)] and used implicitly ten years earlier by the present author and R. R. Phelps [Duke Math. J. 42, 735-750 (1975; Zbl 0332.46013)]. Remember that a topological space X is fragmented by a metric d on X (in general, there is no relation between the topology of X and d) if for any ∅ ≠ A ⊂ X and ϵ > 0 there is a nonempty relatively open subset of A with d- diameter less than ϵ.

Let T be a compact (Hausdorff) space. Then the following are equivalent:
1) T is an RN compact space;
2) T is homeomorphic to a norm-fragmented weak*-compact subset of a dual Banach space;
3) T is homeomorphic to a pointwise compact subset K of [−1, 1]^G, for some set G, such that K A is separable in the uniform topology whenever A is a countable subset of G;
4) there is a super sequentially compact space X and a dense subset D of X such that T is homeomorphic to a norm-bounded subset of (real) C(X) provided with the topology of pointwise convergence on D.

Let K be a RN compact space. Then
1) a closed subspace of K is RN compact;
2) K is sequentially compact;
3) the unit ball of C(K)* is RN compact (in the weak* topology);
4) the space of all Radon probability measures on K is RN compact (in the weak*-topology);
5) K belongs to the Stegall class S;
6) K is metrizable iff it is hereditarily Lindelöf.

The product of countably many RN compact spaces is RN compact.

A compact space is RN compact iff it is fragmented by a l.s.c. metric.

Five open problems are stated.

Reviewer: J.Daneš

MSC:
- 46B20 Geometry and structure of normed linear spaces
- 46B22 Radon-Nikodým, Kreĭn-Milman and related properties
- 46B25 Classical Banach spaces in the general theory

Keywords:
RN compact spaces; fragmentability; norm-fragmented weak*-compact subset of a dual Banach space; Stegall class; hereditarily Lindelöf

Full Text: DOI

References: