Shi, Shuzhong

Nagumo type condition for partial differential inclusions. (English) Zbl 0654.49016

Nonlinear Anal., Theory Methods Appl. 12, No. 9, 951-967 (1988).

The viability problem for autonomous differential inclusions in Hilbert and Banach spaces and for the generalized equation $0 \in F(x)$ is studied. Let V, H be two Hilbert spaces such that $V \subset H = H' \subset V'$, the inclusions being compact and dense, and let $K \subset H$ be a closed set with the so-called internal approximation property. For the differential inclusion (1) $\dot{x} + Ax \in G(x)$, $x(0) = x_0 \in K$, x(t)$\in K$ for all $t \in [0, T]$, where G: $K \to V$ is an upper- semicontinuous multifunction with closed convex values such that G(K) is bounded and $A \in L(V, V')$ is a V-elliptic operator. Under a simple generalization of the so-called tangential condition $[G(x) - Ax] \cap T'_K(x) \neq \emptyset$ for all $x \in K \cap V$, the existence of a $W^{1,2}(0, T)$-solution of problem (1) is proved for all $x_0 \in K$ and any $T > 0$. The proof is based on a finite-dimensional approximation and classical results. Next, under the same assumptions the existence of solutions of the equation Ax$\in G(x)$ in $K \cap V$ is proved.

At the end some finite-difference scheme for equation (1) is proposed, and the existence of a solution of problem (1) in the space $W^\infty(0, T)$ is obtained.

As applications, the boundary and obstacle problems for parabolic differential inclusions, equations and inequalities and for variational inequalities are studied.

Reviewer: Z.Wyderka

MSC:

93B05	Controllability
49J45	Methods involving semicontinuity and convergence; relaxation
35K20	Initial-boundary value problems for second-order parabolic equations
35K85	Unilateral problems for linear parabolic equations and variational inequalities with linear parabolic operators
34A60	Ordinary differential inclusions
34G20	Nonlinear differential equations in abstract spaces
49J40	Variational inequalities

Keywords:

viability; autonomous differential inclusions; tangential condition; obstacle problems; variational inequalities

Full Text: DOI

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