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On extensions of a double covering of plane curves and Weierstrass semigroups of the double covering type. (English) Zbl 1351.14022

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In this paper certain Weierstrass semigroups related to double covering of curves are investigated. For H a numerical semigroup of genus $g = g(H)$ let $d_2(H) := \{s \in \mathbb{N} : 2s \in H\}$, where $\mathbb{N}$ stands for the set of nonnegative integers [*J. C. Rosales* et al., J. Number Theory 103, No. 2, 281–294 (2003; Zbl 1039.20036)]. A semigroup is the *double covering type*, if it is the Weierstrass semigroup $H(\tilde{P})$ at a totally ramified point $\tilde{P}$ of a double covering $\pi : \tilde{C} \to C$ of (projective, irreducible, nonsingular, algebraic) curves over the complex numbers; in this case, $d_2(H(\tilde{P})) = H(P)$ being $P = \pi(\tilde{P})$ [*T. Kato*, Kodai Math. J. 2, 275–285 (1979; Zbl 0425.30038)].

If $g(d_2(H)) \in \{0, 1, 2, 3\}$ and $g(H)$ is large enough, H is the double covering type; see e.g. [*J. Komeda*, J. Reine Angew. Math. 341, 68–86 (1983; Zbl 0498.30053)], [Res. Rep. Kamagawa Inst. Technol. B-33, 37–42 (2009)], [*G. Oliveira* and *F. L. R. Pimentel*, Semigroup Forum 77, No. 2, 152–162 (2008; Zbl 1161.14023)] , [*J. Gilvan de Oliveira* et al., J. Pure Appl. Algebra 214, No. 11, 1955–1961 (2010; Zbl 1194.14048)], [*J. Komeda*, Semigroup Forum 83, No. 3, 479–488 (2011; Zbl 1244.14025)].

Let $\pi : \tilde{C} \to C$, $\tilde{P}$, P be as above. In the paper under review, C is a plane curve of degree $d \geq 4$, T_P stands for the tangent line to C at P. Let M_d be the proposition: π extends to a double covering $\tilde{\pi} : X \to \mathbb{P}^2$ branched along a reduced divisor of degree six containing P. The main result here is a characterization of certain semigroups of the double covering type: (a) If $I_P(T_P \cap C) = d$, then M_d holds if and only if $H(\tilde{P}) = 2H(P) + (6d-1)\mathbb{N}$. (b) Let $I_P(T_P \cap C) = d - 1$ and $T_P \cdot C = (d-1)P + Q$. If $I_Q(T_Q \cap C) = d$, then M_d holds if and only if $H(\tilde{P}) = 2H(P) + \sum_{i-0}^{d-4}(8d - 9 + 2i(d-2))\mathbb{N}$. Particular cases of this result were already computed in [*K. Watanabe*, Semigroup Forum 86, No. 2, 395–403 (2013; Zbl 1285.14036)].

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