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**Intersection cuts for nonlinear integer programming: convexification techniques for structured sets.** (English) [Zbl 1358.90078](#)

*Math. Program.* 155, No. 1-2 (A), 575-611 (2016).

This paper deals with the generalization of split,  $k$ -branch split, and intersection cuts from mixed integer linear programming to the area of mixed integer nonlinear programming. Two techniques are presented that can be used to construct formulas for split,  $k$ -branch split, and general intersection cuts for several classes of convex sets. The authors first introduce an interpolation technique that can be used to construct split and  $k$ -branch split cuts for many classes of sets. Then, the interpolation technique is used to characterize intersection cuts for conic quadratic sets. The authors also introduce an aggregation technique that can be used to construct a wide array of general intersection cuts. The basic principles behind the techniques is presented in a simple, but abstract setting, and then it is utilized to construct more specific cuts to illustrate their power and limitations.

Reviewer: [Samir Kumar Neogy \(New Delhi\)](#)

**MSC:**

[90C10](#) Integer programming  
[90C30](#) Nonlinear programming  
[90C26](#) Nonconvex programming, global optimization  
[90C57](#) Polyhedral combinatorics, branch-and-bound, branch-and-cut

Cited in **2** Reviews  
Cited in **12** Documents

**Keywords:**

[mixed integer nonlinear programming](#); [split](#); [k-branch split](#); [intersection cuts](#); [interpolation technique](#); [aggregation technique](#); [conic quadratic sets](#)

**Software:**

[SCIP](#)

**Full Text:** [DOI](#) [arXiv](#)

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