Sane, Sharad S.; Shrikhande, Mohan S.
Quasi-symmetric 2,3,4-designs. (English) Zbl 0655.05012

Quasi-symmetric designs are block designs with two block intersection numbers $x$ and $y$. In this paper, it is shown that with the exception of $(x, y) = (0, 1)$, for a fixed value of the block size $k$, there are finitely many such designs. Some finiteness results on block graphs are derived. For a quasi-symmetric 3-design with positive $x$ and $y$, the intersection numbers are shown to be roots of a quadratic whose coefficients are polynomial functions of $v, k$ and $\lambda$. Using this quadratic, various characterizations of the Witt-Lüneburg design on 23 points are obtained. It is also shown that if $x = 1$, then a fixed value of $\lambda$ determines at most finitely many such designs. One of the interesting results is as follows: Let $D$ be a quasi-symmetric 3-$(v,k,\lambda)$ design with $x = 1$. Then (i) $v - 2$ divides $k(k - 1)(k - 2)$; (ii) the larger intersection number $y \geq 3$ with equality if and only if $D$ is the unique Witt-Lüneburg 4-design or its residual a 3-design; (iii) $\lambda \geq y$; (iv) $\lambda_3 \geq 4$ with equality if and only if $D$ is the 3-design $(22,7,4)$ design; (v) $\lambda_3 = 5$ if and only if $D$ is the Witt-Lüneburg design on 23 points; (vi) $\lambda_3 \leq k - 2$ with equality if and only if $D$ is the Witt-Lüneburg design on 23 points; (vii) $\lambda_2 \leq v - 2$ with equality if and only if $D$ is the Witt-Lüneburg design on 23 points; (viii) $v$ is bounded below and above as follows,

$$\frac{(k - 2)^2(y - 1)}{y(y - 2)} + \frac{k - 2}{y} - 1 \leq v - 3 \leq \frac{(k - 2)(k - 3)}{y - 2},$$

further, in which the upper bound is sharp with equality if and only if $D$ is the Witt-Lüneburg 4-design on 23 points. Two conjectures on the classification of quasi-symmetric 3-designs are interestingly stated.

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References:


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