Larrión, F.; **Pizaña, M. A.**; **Villarroel-Flores, R.**

On self-clique shoal graphs. (English) Zbl 1333.05226

Discrete Appl. Math. 205, 86-100 (2016).

Summary: The clique graph of a graph G is the intersection graph $K(G)$ of its (maximal) cliques, and G is self-clique if $K(G)$ is isomorphic to G. A graph G is locally H if the neighborhood of each vertex is isomorphic to H. Assuming that each clique of the regular and self-clique graph G is a triangle, it is known that G can only be r-regular for $r \in \{4, 5, 6\}$ and G must be, depending on r, a locally H graph for some $H \in \{P_4, P_2 \cup P_3, 3P_2\}$. The self-clique locally P_4 graphs are easy to classify, but only a family of locally H self-clique graphs was known for $H = P_2 \cup P_3$, and another one for $H = 3P_2$.

We study locally $P_2 \cup P_3$ graphs (i.e. shoal graphs). We show that all previously known shoal graphs were self-clique. We give a bijection from (finite) shoal graphs to 2-regular digraphs without directed 3-cycles. Under this translation, self-clique graphs correspond to self-dual digraphs, which simplifies constructions, calculations and proofs. We compute the numbers, for each $n \leq 28$, of self-clique and non-self-clique shoal graphs of order n, and also prove that these numbers grow at least exponentially with n.

MSC:

05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)

Cited in **2** Documents

Keywords:

clique graphs; self-clique graphs; constant link

Software:

GAP; GENREG; nauty; OEIS

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