Maia, Mariana; Pellegrino, Daniel; Santos, Joedson

An index of summability for pairs of Banach spaces. (English) Zbl 1351.46040

J. Math. Anal. Appl. 441, No. 2, 702-722 (2016).

An m-linear operator $u \in \mathcal{L}(E_1, \dots, E_m; F)$ is called multiple (p,q)-summing if there is a constant $C > 0$ such that

$$\left(\sum_{k_1,\dots,k_m=1}^{n} \left\| u\left(x_{k_1}^{(1)}, \dots, x_{k_m}^{(m)}\right) \right\|^p \right)^{1/p} \leq C \prod_{i=1}^{m} \left\| \left(x_{k_i}^{(i)}\right)_{k_i=1}^{n} \right\|_{w,q},$$

for all $n \in \mathbb{N}$ and for all $x_{k_i}^{(i)} \in E_i$, with $1 \leq k_i \leq n$ and $1 \leq i \leq m$. When an m-linear operator is not multiple (p,q)-summing the above constant C does not exist.

In this interesting paper the authors show that when the above inequality fails, choosing any $n \in \mathbb{N}$, there exists a constant $C_n = C_1 n^s$ that makes the inequality true, for all choices of $x_{k_i}^{(i)} \in E_i$, with $1 \leq k_i \leq n$ and $1 \leq i \leq m$, where the number s depends on m, p and q. More specifically, they define an index $\eta_{(p,q)}^{m-\text{mult}}(E_1, \dots, E_m; F)$ of (non) (p,q)-summability of a pair $(E_1 \times \cdots \times E_m, F)$ as the infimum of numbers $s_{m,p,q} \geq 0$ that satisfy the following property:

There is a constant $C \geq 0$ such that, for every $u \in \mathcal{L}(E_1, \dots, E_m; F)$, for all $n \in \mathbb{N}$ and for all $x_{k_i}^{(i)} \in E_i$, with $1 \leq k_i \leq n$ and $1 \leq i \leq m$,

$$\left(\sum_{k_1,\dots,k_m=1}^{n} \left\| u\left(x_{k_1}^{(1)}, \dots, x_{k_m}^{(m)}\right) \right\|^p \right)^{1/p} \leq C n^{s_{m,p,q}} \prod_{i=1}^{m} \left\| \left(x_{k_i}^{(i)}\right)_{k_i=1}^{n} \right\|_{w,q}.$$

They also define an index for polynomials in the same fashion.

Some of the main presented results are upper bounds, that is the index of summability is always finite, and also upper and lower estimates for the index of summability in some special cases.

Reviewer: Jamilson Ramos Campos (João Pessoa)

MSC:

46G25 (Spaces of) multilinear mappings, polynomials
47L22 Ideals of polynomials and of multilinear mappings in operator theory
47H60 Multilinear and polynomial operators
46B80 Nonlinear classification of Banach spaces; nonlinear quotients

Keywords:

polynomials; multilinear mappings; absolutely summing operators; Banach spaces

Full Text: DOI

References:

[1] Araújo, G.; Pellegrino, D., Optimal Hardy-Littlewood inequalities for \textit{m}-linear forms on ℓ_p spaces with $1 \leq p \leq m$, Arch. Math., 106, 285-295, (2015) · Zbl 1432.32003

[2] Bernardino, A. T.L., On cotype and a Grothendieck-type theorem for absolutely summing multilinear operators, Quaest. Math., 34, 513-519, (2011) · Zbl 1274.46092

[3] Botelho, G.; Pellegrino, D.; Rueda, P., Dominated bilinear forms and 2-homogeneous polynomials, Publ. Res. Inst. Math. Sci., 46, 201-208, (2010) · Zbl 1193.46027

[4] Botelho, G.; Pellegrino, D.; Rueda, P., Cotype and absolutely summing linear operators, Math. Z., 267, 1-7, (2011) · Zbl 1226.47017

[5] Diestel, J.; Jarchow, H.; Tonge, A., Absolutely summing operators, (1995), Cambridge University Press Cambridge · Zbl 0855.47016

[6] Dineen, S., Complex analysis on infinite dimensional spaces, (1999), Springer-Verlag London · Zbl 1034.46504

[7] Galicer, D.; Mansilla, M.; Muro, S., The sup-norm vs. the norm of the coefficients: equivalence constants for homogeneous polynomials

[8] Garling, D. J.H., Inequalities: A journey into linear analysis, (2007), Cambridge University Press Cambridge · Zbl 1135.26014

[9] König, H.; Retherford, J. R.; Tomczak-Jaegermann, N., On the eigenvalues of $(p, 2)$-summing operators and constants associated with normed spaces, J. Funct. Anal., 37, 88-126, (1980) · Zbl 0434.47033

[10] Lindenstrauss, J.; Pelczynski, A., Absolutely summing operators in $\mathcal{L}_p$-spaces and their applications, Studia Math., 29, 275-326, (1968) · Zbl 0183.40501

[11] Matos, M. C., Fully absolutely summing and Hilbert-Schmidt multilinear mappings, Collect. Math., 54, 111-136, (2003) · Zbl 1078.46031

[12] Mujica, J., Complex analysis in Banach spaces, (2010), Dover Publications

[13] Pérez-García, D., Operadores multilineales absolutamente sumantes, (2003), Universidad Complutense de Madrid, Thesis

[14] Pietsch, A., Operator ideals, (1980), North-Holland Publishing Company Amsterdam/New York/Oxford · Zbl 0399.47039

[15] Szarek, S. J., Computing summing norms and type constants on few vectors, Studia Math., 98, 148-156, (1991) · Zbl 0795.46011

[16] Tomczak-Jaegermann, N., Banach-Mazur distances and finite dimensional operator ideals, (1988), Longman, Harlow, and Wiley New York · Zbl 0721.46004