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On trace forms of algebraic number fields. (English) Zbl 0658.10025
Math. Z. 196, 125-127 (1987).

Let L/K be a finite separable field extension and let $Tr_{L/K} < 1 >$ denote the associated trace form on $L \times L$ over K given by $(x, y) \mapsto Tr_{L/K}(xy)$. For any ordering P of K the signature $sign_P(Tr_{L/K} < 1 >)$ is nonnegative. In the paper under review it is shown that for algebraic number fields K the converse is true: Theorem: Let K be an algebraic number field with Witt ring $W(K)$. For any $\phi \in W(K)$ assume $sign_P(\phi) \geq 0$ for all orderings P of K . Then there exists a finite extension L/K such that ϕ is Witt equivalent to $Tr_{L/K} < 1 >$.

This generalizes similar results obtained by Conner and Perlis in the case $K = \mathbb{Q}$. The proof uses a certain version of Hilbert's irreducibility theorem.

Reviewer: [H.-J.Bartels](#)

MSC:

11E16 General binary quadratic forms

12D15 Fields related with sums of squares (formally real fields, Pythagorean fields, etc.)

11E12 Quadratic forms over global rings and fields

Cited in **3** Reviews

Cited in **9** Documents

Keywords:

[symmetric bilinear form](#); [Hilbertian field](#); [separable field extension](#); [trace form](#); [ordering](#); [signature](#); [algebraic number field](#); [Witt ring](#)

Full Text: [DOI](#) [EuDML](#)

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