Phelps, Robert R.
Convex functions, monotone operators and differentiability. (English) ┃Zbl 0658.46035┃
Lecture Notes in Mathematics 1364. Berlin etc.: Springer-Verlag. vii, 115 p. DM 25.00 (1989).

The differentiability properties of convex functions on Banach spaces are related to many branches of mathematics (extreme points, monotone operators, perturbed optimization of real functions, differentiability of vector-valued measure, variational methods). In this book, the author starts from elementary definitions and leads to very deep and recent results. The book is well written and has many usefully examples and exercises. It is a good book both for a graduate course either for reference in convex analysis.

The book consists of seven sections. In the first section, the definitions and classical results (Rademacher's theorem and Mazur's theorem) are considered. The Asplund spaces and the connections between subdifferentials, monotone operators and derivatives of continuous convex functions are studied in the second section. The third section is devoted to establish the classical and recent results on lower semicontinuous convex functions. Two variational principles of Ekland, Borwein and Preiss and their applications in Asplund spaces are considered in sections 3 and 4. Section 5 describes the duality between Asplund spaces and the spaces with the Radon-Nikodym property. In section 6, the author studies the Gâteaux differentiability spaces, which are similar to Asplund spaces (the Fréchet differentiability is replaced by the Gâteaux differentiability). The last section is devoted to the connection between monotone operators and upper semicontinuous set valued mappings having weak compact convex value.

Reviewer: Duong Minh Duc

MSC:

46G05	Derivatives of functions in infinite-dimensional spaces
26B25	Convexity of real functions of several variables, generalizations
46B20	Geometry and structure of normed linear spaces

Cited in **3** Reviews
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Keywords:

differentiability properties of convex functions on Banach spaces; extreme points; monotone operators; perturbed optimization of real functions; differentiability of vector-valued measure; variational methods; Rademacher's theorem; Mazur's theorem; Asplund spaces; subdifferentials; derivatives of continuous convex functions; variational principles of Ekland, Borwein and Preiss; Radon-Nikodym property; Gâteaux differentiability spaces; semicontinuous set valued mappings having weak compact convex value