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Exponential decay of the vorticity in the steady-state flow of a viscous liquid past a rotating body. (English) [Zbl 1342.35244](#)

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Summary: Consider the flow of a Navier-Stokes liquid past a body rotating with a prescribed constant angular velocity, ω , and assume that the motion is steady with respect to a body-fixed frame. In this paper we show that the vorticity field associated to every “weak” solution corresponding to data of arbitrary “size” (*Leray Solution*) must decay exponentially fast outside the wake region at sufficiently large distances from the body. Our result improves and generalizes in a non-trivial way famous results by *D. C. Clark* [Indiana Univ. Math. J. 20, 633–654 (1971; [Zbl 0187.24506](#))] and *K. I. Babenko* and *M. M. Vasil'ev* [J. Appl. Math. Mech. 37, 651–665 (1973); translation from Prikl. Mat. Mekh. 37, 690–705 (1973; [Zbl 0295.76015](#))] obtained in the case $\omega = 0$.

MSC:

[35Q35](#) PDEs in connection with fluid mechanics

[76U05](#) General theory of rotating fluids

[76D05](#) Navier-Stokes equations for incompressible viscous fluids

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