Jung, Da Woon; **Kim, Byung-Ok**; **Kim, Hong Kee**; **Lee, Yang**; **Nam, Sang Bok**; **Ryu, Sung Ju**; **Sung, Hyo Jin**; **Yun, Sang Jo**

On quasi-commutative rings. (English) Zbl 1347.16039

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The authors define a ring R (associative with identity) to be *quasi-commutative* if ab is in the center of R for all $a \in C_{f(x)}$ and $b \in C_{g(x)}$ whenever $f(x)$ and $g(x)$ are in the center of the polynomial ring $R[x]$. Here $C_{h(x)}$ denotes the set of all coefficients of the polynomial $h(x)$. A word of caution; the terminology "quasi-commutative" for rings or for ring elements has already been used many times in many other places with different meanings.

Many examples of quasi-commutative rings are given; in particular also ones that are not commutative. It is shown that this notion is compatible with many ring contructions. For example, a ring R is quasi-commutative if and only if $R[x]$ is quasi-commutative. If $D_n(R)$ denotes the $n \times n$ upper triangular matrix ring with the same element on the diagonal, then it is shown that R is commutative if and only if $D_2(R)$ is commutative which in turn is equivalent to $D_2(R)$ being quasi-commutative. But for $n \geq 3$, $D_n(R)$ is never quasi-commutative. It is also shown that the radicals of the polynomial ring over a quasi-commutative ring have the same behaviour as if over a commutative ring, i.e., for a quasi-commutative ring R, the following ideals coincide: the Jacobson radical of $R[x]$, the Wedderburn radical of $R[x]$, the upper nil radical of $R[x]$, the prime radical of $R[x]$ and the ring of polynomials over the nilradical of R (set of all nilpotent elements of R).

Reviewer: Stefan Veldsman (Port Elizabeth)

MSC:

16U80 Generalizations of commutativity (associative rings and algebras)
16S36 Ordinary and skew polynomial rings and semigroup rings
16N40 Nil and nilpotent radicals, sets, ideals, associative rings
16U70 Center, normalizer (invariant elements) (associative rings and algebras)
16N80 General radicals and associative rings

Keywords:

quasi-commutative rings; polynomial rings; central elements; radicals

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