

Flint, Guy; Hambly, Ben; Lyons, Terry

Discretely sampled signals and the rough Hoff process. (English) Zbl 1348.60058
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Summary: We introduce a canonical method for transforming a discrete sequential data set into an associated rough path made up of lead-lag increments. In particular, by sampling a d -dimensional continuous semimartingale $X : [0, 1] \rightarrow \mathbb{R}^d$ at a set of times $D = \{t_i\}$, we construct a piecewise linear, axis-directed process $X^D : [0, 1] \rightarrow \mathbb{R}^{2d}$ comprised of a past and a future component. We call such an object the Hoff process associated with the discrete data $\{X_t\}_{t_i \in D}$. The Hoff process can be lifted to its natural rough path enhancement and we consider the question of convergence as the sampling frequency increases. We prove that the Itô integral can be recovered from a sequence of random ODEs driven by the components of X^D . This is in contrast to the usual Stratonovich integral limit suggested by the classical Wong-Zakai Theorem [*E. Wong* and *M. Zakai*, *Int. J. Eng. Sci.* 3, 213–229 (1965; [Zbl 0131.16401](#))]. Such random ODEs have a natural interpretation in the context of mathematical finance.

MSC:

- 60G17 Sample path properties
- 60G35 Signal detection and filtering (aspects of stochastic processes)
- 60G44 Martingales with continuous parameter
- 60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)

Cited in 8 Documents

Keywords:

rough path theory; lead-lag path; Hoff process; Wong-Zakai approximations; Itô-Stratonovich correction

Full Text: [DOI](#) [arXiv](#)

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