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Iterated Lie brackets in limit processes in ordinary differential equations. (English)

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A rather general form of the averaging principle [which was announced by the authors in Z. Angew. Math. Phys. 38, 241-256 (1987; Zbl 0616.34004)] is proved. Let m be an integer, $m > 1$, $\phi_i : R \to R$, $i = 1, 2, ..., r$ locally integrable. Denote by $\int \phi_i$ a primitive of ϕ_i, by$\int \phi_j \int \phi_i$ a primitive of $\phi_j \int \phi_i$, by $\int \phi_k \int \phi_j \int \phi_i$ a primitive of $\phi_k \int \phi_j \int \phi_i$ etc. Let $f_j : R^n \to R^n$ be of class $C^{(m)}$, $[f_j, f_k]$ denoting the Lie brackets. Theorem: Let ϕ_j fulfill the following conditions: (1) $\int_s^{s+1} |\phi_i| dt \le C$ for $s \in R$, $i = 1, 2, ..., r$, (2) if $j < m$, $i_1, i_2, ..., i_j \in \{1, 2, ..., r\}$, then the mean value of $\phi_{i_1} \int \phi_{i_2} ... \int \phi_{i_j}$ is zero (i.e. the corresponding primitive is bounded), (3) if $i_1, ..., i_m \in \{1, 2, ..., r\}$, then the mean value of $\phi_{i_1} \int \phi_{i_2} ... \int \phi_{i_m}$ is $\lambda_{i_1 i_2 ... i_m} \in R$. Then the solutions of

$$(4) \quad \dot{x} = f_0(x) + \epsilon^{-\alpha} \sum_{i=1}^{r} f_i(x) \phi_i(\frac{t}{\epsilon}), \quad x(s) = y$$

tend for $\epsilon \to 0$ to the solution of (5) $\dot{x} = f_0(x)$, $x(s) = y$ in case that $\alpha < (m-1)/m$ and to the solution of

$$(6) \quad \dot{x} = f_0(x) + \frac{(-1)^{m-1}}{m} \sum_{i_1, ..., i_m = 1}^{r} [...[f_{i_1}, f_{i_2}]...f_{i_m}] \lambda_{i_1 i_2 ... i_m},$$

x(s)= y in case that $\alpha = (m-1)/m$.

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