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**Iterated Lie brackets in limit processes in ordinary differential equations.** (English)

Zbl 0663.34043

Result. Math. 14, No. 1-2, 125-137 (1988).

A rather general form of the averaging principle [which was announced by the authors in Z. Angew. Math. Phys. 38, 241-256 (1987; Zbl 0616.34004)] is proved. Let  $m$  be an integer,  $m > 1$ ,  $\phi_i : R \rightarrow R$ ,  $i = 1, 2, \dots, r$  locally integrable. Denote by  $\int \phi_i$  a primitive of  $\phi_i$ , by  $\int \phi_j \int \phi_i$  a primitive of  $\phi_j \int \phi_i$ , by  $\int \phi_k \int \phi_j \int \phi_i$  a primitive of  $\phi_k \int \phi_j \int \phi_i$  etc. Let  $f_j : R^n \rightarrow R^n$  be of class  $C^{(m)}$ ,  $[f_j, f_k]$  denoting the Lie brackets. Theorem: Let  $\phi_j$  fulfill the following conditions: (1)  $\int_s^{s+1} |\phi_i| dt \leq C$  for  $s \in R$ ,  $i = 1, 2, \dots, r$ , (2) if  $j < m$ ,  $i_1, i_2, \dots, i_j \in \{1, 2, \dots, r\}$ , then the mean value of  $\phi_{i_1} \int \phi_{i_2} \dots \int \phi_{i_j}$  is zero (i.e. the corresponding primitive is bounded), (3) if  $i_1, \dots, i_m \in \{1, 2, \dots, r\}$ , then the mean value of  $\phi_{i_1} \int \phi_{i_2} \dots \int \phi_{i_m}$  is  $\lambda_{i_1 i_2 \dots i_m} \in R$ . Then the solutions of

$$(4) \quad \dot{x} = f_0(x) + \epsilon^{-\alpha} \sum_{i=1}^r f_i(x) \phi_i\left(\frac{t}{\epsilon}\right), \quad x(s) = y$$

tend for  $\epsilon \rightarrow 0$  to the solution of (5)  $\dot{x} = f_0(x)$ ,  $x(s) = y$  in case that  $\alpha < (m-1)/m$  and to the solution of

$$(6) \quad \dot{x} = f_0(x) + \frac{(-1)^{m-1}}{m} \sum_{i_1, \dots, i_m=1}^r [\dots [f_{i_1}, f_{i_2}] \dots f_{i_m}] \lambda_{i_1 i_2 \dots i_m},$$

$x(s) = y$  in case that  $\alpha = (m-1)/m$ .

Reviewer: J.Kurzweil

**MSC:**

34C29 Averaging method for ordinary differential equations

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**References:**

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