Enflo, Per

On the invariant subspace problem for Banach spaces. (English) Zbl 0663.47003

Acta Math. 158, 213-313 (1987).

Since the very beginning of the study of linear operators in infinite dimensional vector spaces one of the basic problems was to find decompositions of the operators in simpler parts, more or less analogous to Jordan cells for finite matrices. In particular cases, such as for normal operators in Hilbert space, compact operators in any Banach space, and some other important classes of operators, such "Jordan type" decompositions were established and intensively studied. These decompositions are based upon, or furnishing, non-trivial invariant subspaces for the respective operators. But the problem whether for every operator on a Banach (or even Hilbert) space there exist a non-trivial invariant subspace remained longtime unsettled.

It was P. Enflo who first outlined (1975/76) a way to construct an operator T on some (non-reflexive) Banach space for which no non-trivial invariant subspace exists.

The basic idea is to start with the linear space of polynomials p in one variable x and try to find a concept of norm in this space such that the operator of multiplication by x in the norm-completion B of this space have only non-trivial invariant subspaces, or equivalently, that for every $h \in B$ there exist polynomials p such that ph be as close to the constant function 1 in B as we wish.

How natural this approach should appear, its realization demands various deep and complex means. It is no surprise that elaboration (and multiple controlling) of all steps had to take several years. Thus, the paper of Enflo, expounding his proof has been presented to the editors only on Febr. 10, 1981, and in a final version on May 21, 1985, while the printing is dated July 28, 1987.

In the meantime, *B. Beauzamy* has published a paper in the journal Integral Equations and Operator Theory 8, 314–384 (1985; Zbl 0571.47002), which, by also using ideas and techniques due to Enflo, arrived in a different and somewhat simpler way to the same (or even slightly stronger) result. In between, *C. Read* had also constructed an operator without non-trivial invariant subspace, by another, rather involved combinatorical arguments, which turned out to hold even for the concrete case $B = \ell^1$ [Bull. London Math. Soc. 16, 337–401 (1984; Zbl 0566.47003), 17, 305-317 (1985; Zbl 0574.47006) and J. London Math. Soc., II. Ser. 34, 335–348 (1986)].

All these papers are deep and ingenious pieces of mathematical creation, which certainly will act as incentives in further areas of mathematical research also.

As a sample let us mention just one of the side-effects of Enflo's reasonings: If A, B are polynomials in several variables, then $|AB| \geq K|A||B|$, where $|A|$ is the sum of the absolute values of the coefficients of A, and K is a constant depending on the degrees of the polynomials A and B, but not on the number of the variables.

Reviewer: Béla Szökefalvi-Nagy

MSC:

47A15 Invariant subspaces of linear operators
46B20 Geometry and structure of normed linear spaces

Keywords:

Banach space operator; Jordan type decompositions; invariant subspaces

Full Text: DOI

References:

[1] Beauzamy, B., Sous-espaces invariant de type fonctionel dans les espaces de Banach.Acta Math., 144 (1980), 65–82. · Zbl 0438.47005 · doi:10.1007/BF02392121

[2] Enflo, P., On the invariant subspace problem for Banach spaces.Seminaire Maurey-Schwarz (1975–76).

[3] Enflo, P.,On the invariant subspace problem for Banach spaces. Institute Mittag-Leffler report no. 9 (1980). · Zbl 0663.47003