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Lebesgue constants of the Walsh system and Banach limits. (English. Russian original)

Zbl 1354.42047

Sib. Math. J. 57, No. 3, 398-410 (2016); translation from Sib. Mat. Zh. 57, No. 3, 512-526 (2016).

The authors make a careful analysis of the Lebesgue constants for W_k, the Walsh system in $[0,1]$, given by $L_n(W) = \int_0^1 |\sum_{k=1}^n W_k(t)|dt$, $n \in \mathbb{N}$. Refining some estimates due to *N. J. Fine* [Trans. Am. Math. Soc. 65, 372–414 (1949; Zbl 0036.03604)], they manage to compute $\max_{1\leq n \leq 2^{2m+1}} L_n(W)$ for $m \in \mathbb{N}$, which allows them, using a result by *G. G. Lorentz* [Acta Math. 80, 167–190 (1948; Zbl 0031.29501)], to get that the sequence $\{\frac{L_n(W)}{\log_2 n}\}$ is not almost convergent.

They also consider the step functions $f_n(t) = \frac{1}{n} L_{[2^n(1+t)]}(W)$ and show that $\lim_{n\to\infty} f_n(t) = \frac{1}{4}$ for almost all $t \in [0,1]$, $\lim_{n\to\infty} f_n(t) = 0$ for all dyadic rational $t \in [0,1]$ and that there exists a dense set $A \subset [0,1]$ such that $\liminf_{n\to\infty} f_n(t) = 0$ and $\limsup_{n\to\infty} f_n(t) = \frac{1}{3}$ for $t \in A$.

Reviewer: Oscar Blasco (Valencia)

MSC:

42C10 Fourier series in special orthogonal functions (Legendre polynomials, Walsh functions, etc.)

Cited in **2** Documents

Keywords:

Walsh functions; Rademacher functions; Lebesgue constants; Banach limit; almost convergent sequence

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