zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Alper, J.; **Isaev, A. V.**; **Kruzhilin, N. G.**
Associated forms of binary quartics and ternary cubics. (English) Zbl 1372.14038
Transform. Groups 21, No. 3, 593-618 (2016).

The main object of the paper under review is the complex vector space Q_n^d of n-ary forms of degree d with complex coefficients ($n \geq 2$, $d \geq 3$). More precisely, the authors are interested in the map Φ sending a nondegenerate n-ary d-form f to the associated form which is an element of the space dual to $Q_n^{n(d-2)}$. Their focus is on the cases of binary quartics and ternary cubics, the only cases when Φ preserves the degree. They show that in each of these cases the projectivization of Φ induces an equivariant (with respect to an action of SL_n) involution on the projectivization of the space of nondegenerate forms with one orbit removed. Furthermore, they show that a such a nontrivial rational equivariant involution is unique. In particular, Φ yields a unique equivariant involution on the space of elliptic curves with nonvanishing j-invariant. They give a simple interpretation of this involution in terms of projective duality and express it via classical contravariants, in the spirit of Cayley and Sylvester.

Eventual applications in singularity theory are also explained.

Reviewer: Boris Kunyavskii (Ramat Gan)

MSC:

14L24 Geometric invariant theory
11E20 General ternary and quaternary quadratic forms; forms of more than two variables
14H52 Elliptic curves
32S10 Invariants of analytic local rings
32S25 Complex surface and hypersurface singularities

Keywords:
geometric invariant theory; binary quartic; ternary cubic; elliptic curve; contravariant

Full Text: DOI arXiv

References:

[1] J. Alper, A. V. Isaev, \textit{Associated forms in classical invariant theory and their applications to hypersurface singularities}, Math. Ann. \textbf{360} (2014), 799-823. · Zbl 1308.14048

[2] J. Alper, A. Isaev, \textit{Associated forms and hypersurface singularities: the binary case}, preprint, http://arxiv.org/abs/1407.6838. · Zbl 1308.14048

[3] H. Bass, \textit{On the ubiquity of Gorenstein rings}, Math. Z. 82 (1963), 8-28. · Zbl 0112.26604

[4] A. Cayley, \textit{On the 34 concomitants of the ternary cubic}, Amer. J. Math. 4 (1881), 1-15. · Zbl 13.0107.01

[5] L. Di Biagio, E. Postinghel, \textit{Apolarity, Hessian and Macaulay polynomials}, Comm. Algebra \textbf{41} (2013), 226-237. · Zbl 1262.14069

[6] I. Dolgachev, \textit{Classical Algebraic Geometry. A Modern View}, Cambridge University Press, Cambridge, 2012. · Zbl 1252.14001

[7] M. G. Eastwood, \textit{Moduli of isolated hypersurface singularities}, Asian J. Math. \textbf{8} (2004), 305-313. · Zbl 1084.32019

[8] M. G. Eastwood, A. V. Isaev, \textit{Extracting invariants of isolated hypersurface singularities from their moduli algebras}, Math. Ann. \textbf{356} (2013), 73-98. · Zbl 1277.32031

[9] J. Elias, M. E. Rossi, \textit{Isomorphism classes of short Gorenstein local rings via Macaulay's inverse system}, Trans. Amer. Math. Soc. \textbf{364} (2012), 4589-4604. · Zbl 1281.13015

[10] E. B. Elliott, \textit{An Introduction to the Algebra of Quantics}, Oxford University Press, 1895.

[11] J. Emsalem, \textit{Géométrie des points épais}, Bull. Soc. Math. France \textbf{106} (1978), 399-416. · Zbl 0396.13017

[12] I. M. Gelfand, M. M. Kapranov, A. V. Zelevinsky, \textit{Discriminants, Resultants and Multidimensional Determinants}, Modern Birkhäuser Classics, Birkhüser Boston, Boston, MA, 2008. · Zbl 1138.14001

[13] U. Görtz, T. Wedhorn, \textit{Algebraic Geometry} I. \textit{Schemes with Examples and Exercises}, Advanced Lectures in Mathematics, Vieweg + Teubner, Wiesbaden, 2010. · Zbl 1213.14001

[14] G.-M. Greuel, C. Lossen, E. Shustin, \textit{Introduction to Singularities and Deformations}, Springer Monographs in Mathematics, Springer, Berlin, 2007. · Zbl 1125.32013

[15] J. Harris, \textit{Algebraic Geometry. A First Course}, Graduate Texts in Mathematics, Vol. 133, Springer-Varlag, New York, 1992. · Zbl 0779.14001

[16] C. Huneke, \textit{Hyman Bass and ubiquity: Gorenstein rings}, in: \textit{Algebra, K-Theory, Groups, and Education} (New York, 1997), Contemp. Math., Vol. 243, Amer. Math. Soc., Providence, RI, 1999, pp. 55-78. · Zbl 0960.13008

[17] A. Iarrobino, \textit{Associated Graded Algebra of a Gorenstein Artin Algebra}, Mem. Amer. Math. Soc., Vol. 514, Amer. Math. Soc., Providence, RI, 1994. · Zbl 0793.13010

[18] V. Lakshmibai, K. N. Raghavan, \textit{Standard Monomial Theory. Invariant Theoretic approach}, Encyclopaedia of Mathematical Sciences, Vol. 137, Subseries Invariant Theory and Algebraic Transformation Groups, Vol VIII, Springer, Berlin, 2008. · Zbl 1137.14036

[19] F. S. Macaulay, \textit{The Algebraic Theory of Modular Systems, reprint of the 1916 original}, Cambridge University Press, Cambridge, 1994.

[20] J. Mather, S. S.-T. Yau, \textit{Classification of isolated hypersurface singularities by their moduli algebras}, Invent. Math. \textbf{69} (1982), 243-251. · Zbl 0499.32008

[21] W. P. Milne, D. G. Taylor, \textit{Relation between apolarity and the Pippian-Quippian syzygetic pencil}, Proc. London Math. Soc. \textbf{S2-20} (1922), 101-106. · Zbl 48.0733.02

[22] S. Mukai, \textit{An Introduction to Invariants and Moduli}, Cambridge Studies in Advanced Mathematics, Vol. 81, Cambridge University Press, Cambridge, 2003. · Zbl 1033.14008

[23] D. Mumford, \textit{Geometric Invariant Theory}, Ergebnisse der Mathematik und ihrer Grenzgebiete, Neue Folge, Band 34, Springer-Verlag, Berlin, 1965.

[24] K. Saito, \textit{Einfach-elliptische Singularitäten}, Invent. Math. \textbf{23} (1974), 289-325. · Zbl 0296.14019

[25] A. H. W. Schmitt, \textit{Geometric Invariant Theory and Decorated Principal Bundles}, Zürich Lectures in Advanced Mathematics, European Mathematical Society, Zürich, 2008. · Zbl 1159.14001

[26] А. Н. Шошитайшвили, \textit{О Функции с изоморфными якобиевыми идеалами}, Функц. анализ и его прил. \textbf{10} (1976), вып 2, 57-62. Engl. transl.: A. N. Shoshitaĭshvili, \textit{Functions with isomorphic Jacobian ideals}, Funct. Anal. Appl. \textbf{10} (1976), no. 2, 128-133. · Zbl 0267.17003

[27] R. P. Stanley, \textit{Hilbert functions of graded algebras}, Adv. Math. \textbf{28} (1978), 57-83. · Zbl 0384.13012

[28] J. J. Sylvester, \textit{A synoptical table of the irreducible invariants and covariants to a binary quintic, with a scholium on a theorem in conditional hyperdeterminants}, Amer. J. Math. \textbf{1} (1878), 370-378. · Zbl 10.0089.01

[29] E. A. Tevelev, \textit{Projectively dual varieties}, J. Math. Sci. (N.Y.) \textbf{117} (2003), 4585-4732. · Zbl 1050.14048